Indian Maritime University

(A Central University, Govt of India)

End Semester Examinations – December 2023

Programme Name: B Tech (ME)

Semester: II

Subject Code: UG11T4201

Subject Name: Mathematics II

Date: 13.11.2023	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

General Instructions

(i) All Sections (A, B & C) are to be attempted.
(ii) Options, if any, are specified in respective section.
(iii) Use of approved type Scientific Calculator is permitted.

Section A

Fill in the Blanks OR Choose the correct answer as applicable. [10X1=10 Marks]

1.If f(x) is an even function in (-l, l) then the value of b_n =__________

A. 1
B. π
C. 0
D. $\frac{\pi}{2}$

2. Half range Cosine series of f(x) in (0, c) is

A. f(x) = = $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{c}$

B. f(x) = = $\frac{1}{2}a_0$

C. f(x) = = $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{c}$

D. None of above

3. L(cos at) =______________

A. $\frac{s}{s^2+a^2}$, (s > 0)

B. $\frac{1}{s^2+a^2}$, (s > 0)

C. $\frac{s}{s^2-a^2}$, (s > 0)

D. $\frac{s}{s^2+a^2}$, (s > 0)

4. The partial differential equation

$\frac{\partial^2 u}{\partial t^2} - C^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$; where $c \neq 0$ is known as

a) Wave equation
b) 1-dimensional heat equation
c) 2-dimensional heat equation
d) Laplace equation

5. The order and degree of given differential equation $\frac{d^2y}{dx^2} + y\left(\frac{dy}{dx}\right)^3 + e^{3x} = 0$ is__________

A. order 1 degree 3.
B. order 2 degree 3
C. order 2 degree 1
D. order 1 degree 2

6. What is the general solution of the given differential equation?

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$$

A. $x = (C_1 + C_2 t)e^{-3t}$

B. $x = (C_1 + C_2)e^{-3t}$

C. $x = C_1 + C_2 t e^{-3t}$

D. $x = C_1 + C_2 e^{-3t}$

7. The inverse Laplace transform of 1/s is

a) t
b) t^2
c) e^{-t}
d) 1

8. The singular point of the function $\frac{1}{4z - z^2}$ are

A. $z = 0$ *and* $z = -4$

B. $z = 0$ *and* $z = 4$

C. $z = 4$ *and* $z = -4$

D. $z = 0$ *and* $z = 2$

9. Residue of $\frac{\cos z}{z}$ at $z = 0$ is

A. 1
B. -1
C. 2
D. 0

10. The integrating factor of the given differential equation is

$$\frac{dx}{dy} + \frac{3x}{y} = \frac{1}{y^2}$$

A. e^{y^3}

B. y^3

C. x^3

D. $-y^3$

Section B

Answer the following
[5x2 = 10 Marks]

11. Obtain the coefficient a_0 of Fourier series for $f(x)= e^x$ in the interval $0<x<2\pi$.

12. Solve $\frac{dy}{dx} = e^{-2y}\,(e^{3x} + x^2)$

13. Find the Laplace transforms of $e^{2t} \cos 4t$

14. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\,\frac{dy}{dx} - 6y = 0$

15. Evaluate using Cauchy's Integral formula $\oint \frac{3z^2+7z+1}{(z-1)}\,dz$,

where c is the circle $|z| = 1/2$.

Section C

Answer any 5 out of 7 questions.

16. a) Express $f(x)=x/2$ as a Fourier series in the interval $(-\pi, \pi)$. (4)

b) Find the Fourier series of $f(x)=x^2$ in the interval $(0, 2\pi)$.

Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$ (6)

17. a) Solve $(x^4 - 2xy^2 + y^4)\,dx - (2x^2y - 4xy^3 + \sin y)\,dy = 0$ (5)

b) Solve $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ (5)

18. a) Solve $(D^2+3D+2)\,y = \sin 2x$ (5)

b) Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ (5)

19. a) Evaluate $\int_0^\infty e^{-3t}\, t^5 dt$ (5)

b) Find $L^{-1}\left[\frac{1}{\{(s-1)(s+2)(s+1)\}}\right]$ (5)

20. a) Find Inverse Laplace transforms of $\frac{1}{s^2(s+5)}$ (5)

b) Using Convolution theorem evaluate $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$ (5)

21. a) Evaluate $\int_c \frac{(z+3)}{(z+1)(z-2)}\, dz$ where c is the circle $|z| = 3$. (5)

b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $|z| < 1$ using Taylor's series (5)

22. a) Solve the given partial differential equation by direct integration method
$\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ (5)

b) Form the partial differential equation by eliminating the arbitrary constants from $(x-a)^2 + (y-b)^2 + z^2 = c^2$ (5)