

INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)
End Semester Examination Dec 2019/Jan 2020
B.Tech (Marine Engineering)
Semester -I
UG11T1102/UG11T2102- Mathematics -I

Date: 12.12.2019

Max Marks: 70

Time: 3 Hours

Pass Marks: 35

Note: i. Use of approved type of scientific calculator is permitted.
ii. The symbols have their usual meanings.

Part-A
(All Questions are Compulsory)

(2x10=20 Marks)

1. Find the n^{th} derivative of $e^{2x} \cos^2 x \sin x$
2. Find the Radius of Curvature at $x = \frac{\pi}{2}$ on the curve $y = 4 \sin x - \sin 2x$
3. If $u = x \log xy$, where $x^3 + y^3 + 3xy = 1$, find $\frac{du}{dx}$
4. If $u = x^y$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$
5. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$
6. Show that $\int_0^{\infty} \frac{x^c}{c^x} dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}$
7. Find the value of a so that the vector $\vec{F} = (ax + 3y + 4z)\vec{i} + (x - 2y + 3z)\vec{j} + (3x + 2y - z)\vec{k}$ is solenoidal.
8. Find the extremals of the functional $\int_{x_0}^{x_1} \frac{y'^2}{x^3} dx$.
9. Test the analyticity of the function $f(z) = z^2$
10. Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2) dx dy$

Part -B (10 x5=50 Marks)
(Answer any 5 of the following)

11. a) If $y^{1/m} + y^{-1/m} = 2x$ prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$
[5 marks]
- b) Find the asymptotes of the curve
 $y^3 - 2xy^2 - x^2y + 2x^3 + 3y^2 - 7xy + 2x^2 + 2y + 2x + 1 = 0$ [5 marks]
12. a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ [5 marks]
- b) Examine the following function for extreme values
 $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. [5 marks]

13. a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 = z - 3$ at the point $(2, -1, 2)$ [5 marks]
- b) Prove that $\nabla r^n = nr^{n-2}\vec{r}$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ [5 marks]
14. a) Investigate the values of λ and μ so that the equation
- $$x + y + z = 6$$
- $$x + 2y + 3z = 10$$
- $$x + 2y + \lambda z = \mu$$
- Have
- (i) no solution
- (ii) infinite number of solution
- (iii) unique solution [5 marks]
- b) Find the Eigen values and Eigen vectors of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ [5 marks]
15. a) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. [5 marks]
- b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx dy dz$ [5 marks]
16. a) Find analytic function whose imaginary part is $e^x \sin y$ [5 marks]
- b) Evaluate $\int_C \frac{e^z}{(z-1)(z-2)} dz$, where $C : |z| = 3$ [5 marks]
17. a) Find the curves on which the functional $\int_0^1 [y'^2 + 12xy] dx$ with $y(0) = 0$ and $y(1) = 1$ [5 marks]
- b) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ [5 marks]
