Indian Maritime University

(A Central University, Govt. of India)

End Semester Examinations: June 2023

Semester: II

Programme Name – B Tech (ME)

Subject Code: UG11T4201

Subject Name: Mathematics - II

Date: 26.05.2023	Max. Marks: 70
Duration: 03 Hrs.	Pass Marks: 35

General Instructions

(i) All sections (A,B & C) are to be attempted

(ii) Options, if any, are specified in respective sections

Section A

Choose the correct answer as applicable **[10 x 1 = 10 Marks]**

Q1. P. I. of $(D^2 + 4)y = cos\ 2x$ is .....

A) $\frac{1}{2} sin\ 2x$

B) $\frac{1}{2} x\ \sin\ 2x$

C) $\frac{1}{4} x\ \sin\ 2x$

D) $\frac{1}{2} x\ \cos\ 2x$

Q2. If $L\{f(t)\} = f(s)$, then $L\{e^{-at} f(t)\}$ is.....

A) $f(s-a)$

B) $f(s+a)$

C) $f(s)$

D) None of these

Q3. Laplace transform of $t^4 e^{-at}$ is.....

A) $\frac{4!}{(s+a)^4}$

B) $\frac{4!}{(s+a)^5}$

C) $\frac{4!}{(s-a)^4}$

D) $\frac{5!}{(s-a)^5}$

Q4. The necessary and sufficient condition for differential equation $M\,dx + N\,dy = 0$ to be exact is

A) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$

B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$

C) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

D) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

Q5. The integrating factor of the differential equation $\frac{dx}{dy} + \frac{3x}{y} = \frac{1}{y^2}$ is

A) e^{y^3}

B) y^3

C) x^3

D) $-y^3$

Q6. Degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^5 - x^2y = 0$ ..... is

A) 2

B) 0

C) 3

D) 5

Q7. The period of $cos\ 3x$ is x = ...

A) $2\pi/3$

B) π

C) 0

D) None of these

Q8. Fourier expansion of an odd function has only _____ terms

A) cosine

B) both sine and cosine

C) sine

D) None of these

Q9. $\cos h^2 x - sinh^2 x =$ _____?

A) 0

B) 1

C) 2

D) π

Q10. The expression for Laplace's equation in two dimensions is

A) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

B) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$

C) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

D) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 1$

Section B

Short Answer Type Questions **[2 x 5 = 10 Marks]**

Q11. Find the complementary function of $\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = xe^{3x}$

Q12. In the Fourier series expansion of $f(x) = x \sin x$ in the interval $0 < x < 2\pi$ find the value of a_1.

Q13. Solve $\frac{\partial^2 z}{\partial y \partial x} = \frac{x}{y} + a$

Q14. Verify if $f(z) = 2x.y + i(x^2 - y^2)$ is analytic or not.

Q15. Find the P.I of $(D^2 + 5D + 6)y = e^x$

Section C

Seven Questions of 10 Marks each of which any 5 questions to be answered.

Q16. (i) Find the differential equation of a family of circles passing through origin and having centres on x-axis. (5)

(ii) Solve the following differential equation $(x^2 - y^2)dx = 2xy\,dy$ (5)

Q17. (i) Solve the differential equation (5)

$$(x^2 - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$$

(ii) Find the orthogonal trajectories of the family of Parabolas (5)

$y = a\,x^2$

Q18. (i) Solve $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 13x = 0$ given $x(0) = 0$, $\frac{dx}{dt}(0) = 2$ (5)

(ii) Find P.I of $\frac{d^2y}{dx^2} - y = e^x + x^2e^x$ (5)

Q19. (i) Find the Laplace transforms of $e^{-3t}\,sin\;5t\,.\,sin\,3t$ (5)

(ii) Find the inverse Laplace transforms of $\frac{1}{S^2 - 5S + 6}$ (5)

Q20. (i) Expand the function $f(x) = x\,sin\,x$ as a Furier series in the interval $-\pi \leq x \leq \pi$ (7)

(ii) Find a_0 in the Fourier series expansion of $f(x) = x - x^2$ in the interval from $x = -\pi$ *to* $x = \pi$ (3)

Q21. (i) Evaluate $\oint_C \frac{e^z dz}{z^2 + 4}$, $C : |z - i| = 2$ (5)

(ii) Evaluate $\oint_C \frac{3z^2 + 2}{(z-1)(z^2+9)}\,dz$, $C : |z - 2| = 2$ (5)

Q22. (i) Form the partial differential equation (by eliminating the arbitrary constants) $(x - a)^2 + (y - b)^2 = z^2\,cot^2\,\alpha$ (5)

(ii) Solve the equation by the method of Separation of variables (5)

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = 0$$

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