

Indian Maritime University
(A Central University, Govt. Of India)
B. Tech(Marine Engineering)
Semester-II
End Semester Examination
Mathematics-II
(UG11T3202)

Time: 3 Hours
Date: 21.06.2022

Max Marks: 70
Pass Marks: 35

Part- A, Q.No. 1.
(10x01=10 Marks)

(i). Solution of $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

(i) $\frac{y-x}{1+xy} = c$ (ii) $\frac{y+x}{1+xy} = c$ (iii) $\frac{y-xy}{1+xy} = c$ (iv) $\frac{y-x}{1-xy} = c$

(II). Complementary Function of $(D^2 + 2D + 1)y = 0$

(i) $(c_1 + c_2x)e^{-x}$ (ii) $c_1e^x + c_2e^{-2x}$ (iii) $(c_1 + c_2x)e^{-3x}$ (iv) $(c_1 + c_2x)e^{-2x}$

(III). Laplace Transform of t^3e^{-3t} is

(i) $\frac{6}{(s-3)^4}$ (ii) $\frac{2}{(s-3)^4}$ (iii) $\frac{6}{(s+3)^4}$ (iv) $\frac{3}{(s-3)^4}$

(IV). Inverse Laplace of $\frac{1}{\sqrt{(s+a)}}$ is

(i) $\frac{e^{-at}}{\sqrt{\pi t}}$ (ii) $\frac{e^{-2at}}{\sqrt{\pi t}}$ (iii) $\frac{e^{-at}}{\sqrt{2\pi t}}$ (iv) $\frac{e^{-3at}}{\sqrt{\pi t}}$

(V). If $f(x) = x \sin x$, $0 < x < 2\pi$ then a_0 is equal to

(i) 2 (ii) -2 (iii) -3 (iv) 3

(VI). If $f(x) = x^2 - 2$, when $-2 \leq x \leq 2$, b_n is equal to

(i) 1 (ii) 0 (iii) 2 (iv) -1

(VII). Laplace transform of $e^t u(t-1)$ is equal to

(i) $\frac{e^{(s-1)}}{(s-1)}$ (ii) $\frac{e^{-(s-1)}}{(s-1)}$ (iii) $\frac{e^{(s+1)}}{(s-1)}$ (iv) $\frac{e^{(s-1)}}{(s+1)}$

(VIII). The chance that a leap year selected at random will contains 53 Sundays is

(i) $\frac{2}{7}$ (ii) $\frac{3}{7}$ (iii) $\frac{4}{7}$ (iv) $\frac{5}{7}$

(IX). If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$,

Then $P(A \cup B)$ is equal to

(i) $\frac{1}{5}$ (ii) $\frac{2}{5}$ (iii) $\frac{3}{5}$ (iv) $\frac{4}{5}$

(X). Two man hit at a target with probabilities $\frac{1}{2}$ and $\frac{1}{3}$ respectively. What is the probability that exactly one of them hits the target ?

(i) $\frac{1}{2}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{6}$ (iv) $\frac{2}{3}$

Part - B, Q.No. 2

Answer the following (5x2=10 Marks)

- (i). Find a₀ Fourier series to represents $x - x^2$ from $x = -\pi$ to $x = \pi$.
 (II). Find the Laplace transform of $f(t) = e^{2t} \cos^2 t$.
 (III). Find the inverse Laplace transform of $\log \frac{s+1}{s-1}$.
 (IV). Solve $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0$.
 (V). Find probability density function of a variance X is

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$.

Part - C

Answer any 5 out of 7 questions (5 x 10= 50 marks)

- 3.(a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$. (5)
 (b) Obtain the Fourier expansion of $x \sin x$ as cosine series in $(0, \pi)$. (5)
 Hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots \infty = \frac{\pi-2}{4}$.
- 4.(a) Evaluate $\int_0^\infty te^{-2t} \sin t dt$. (5)
 (b) Find the inverse Laplace transforms of $\frac{s}{(s^2+1)^2}$. (5)
- 5.(a) Solve $\frac{dz}{dx} + \left(\frac{z}{x}\right) \log z = \frac{z}{x} (\log z)^2$. (5)
 (b) Solve by method of variation of parameters $y'' - 6y' + 9y = \frac{e^{3x}}{x}$. (5)
- 6.(a) In a bolt Factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 1, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B ?
 (b) Find the moment generating function of the random variable whose moments are $\mu'_r = (r + 1)! 2^r$. (5)
7. (a)(i) Out of 400 families with 5 children each, how many would you expect to have (ii) 3 boys (iii) 5 girls (iv) either 2 or 3 boys ? Assume equal probabilities for boys and girls. (5)
 (b) An insurance company finds that 0.005% of the population dies from a certain kind of accident each year. What is the probability that the company must pay off no more than 3 of 10,000 insured risks against such incident in a given year ? (5)
8. (a) Find the Fourier series to represent x^2 in the interval $(-1, 1)$. (5)
 (b) Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$. (5)
9. (a) Apply convolution theorem to evaluate $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$. (5)
 (b) Use transform method to solve $x'' - 2x' + x = e^t$ with $x = 2$
 $x' = -1$ at $t = 0$. (5)