

INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

End Semester Examinations December 2018

B. Tech (Marine Engineering)

Semester – I

MATHEMATICS-I (UG11T2102)

Date: 29-12-2018

Maximum Marks: 100

Time: 3Hrs

Pass Marks: 50

- Note:** i. Use of approved type of scientific calculator is permitted.
ii. The symbols have their usual meanings.

Part – A

Marks: 10 x 3 = 30

(All Questions are compulsory)

1. a. State Leibnitz's theorem and find the n^{th} derivative of $y = e^x(2x + 3)^3$
- b. What is the formula for radius of curvature and centre of curvature at any point $P(x, y)$ on the curve $y = f(x)$
- c. Find the equations of the tangent plane and the normal to the surface $2x^2 + y^2 + 2z - 3$ at $(2, 2, -3)$
- d. Evaluate by $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$ $\alpha \geq 0$ differentiation under the integral sign.
- e. What is the area of the region in the first quadrant bounded by the x axis and the curve $y = \sin x$.

Corrections received

- f. Find the volume of a sphere of radius a , using integration.
- g. If $u = x^2 + y^2 + z^2$ and $\vec{V} = x\vec{i} + y\vec{j} + z\vec{k}$, show that $\text{div}(u\vec{V}) = 5u$.
- h. Given $\vec{A} = 2\vec{i} + 2\vec{j} - \vec{k}$, $\vec{B} = 6\vec{i} - 3\vec{j} + 2\vec{k}$ find the unit vector perpendicular to both \vec{A} and \vec{B} .
- i. Find the Eigen values of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$
- j. Evaluate by Cauchy's integral formula $\int_C \frac{z^2 - z + 1}{z - 1} dz$ where C is the circle $|z| = 1$.

Part - B Marks : 5 x 14 = 70
(Answer any 5 of the following 7 Questions)

(Note for Question Paper setters if necessary)

2. a. If $y = \sin \log(x^2 + 2x + 1)$ prove that
- $$(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$$
- b. Find the radius of curvature for the rectangular hyperbola $xy = c^2$.
- (7+7 = 14)
3. a. The sum of 3 number is constant. Prove by Lagrange multiplier method their product is maximum when they are equal.

b. If $u = \frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y}$

show that $\frac{x\partial u}{\partial x} + \frac{y\partial u}{\partial y} + \frac{z\partial u}{\partial z} = 0$

(7+7 = 14)

4. a. Find the area of the segment cut off from the parabola $x^2 = 8y$ by the line $-2y + 8 = 0$.

b. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(7+7 = 14)

5. a. Evaluate $\int \int xy(x+y) dx dy$ over the area between $y = x^2$ and $y = x$.

b. Prove $\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2} \log(1+a)$ where $a \geq 0$ by

differentiation under the integral sign.

(7+7 = 14)

6. a. Using the line integral, compute the work done by the force $f = (2y + 3)\bar{i} + xz\bar{j} + (yz - x)\bar{k}$ when it moves a particle from the point $(0, 0, 0)$ to the point $(2, 1, 1)$ along the curve $x = 2t^2, y = t, z = t^3$.

b. If \vec{A} and \vec{B} are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

(7+7 = 14)

7. a. Using Cayley Hamilton theorem, find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

b. Using Cayley Hamilton theorem that find the inverse
of $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

(7+7 = 14)

8. a. Use Cauchy's integral formula to calculate

$$\int_C \frac{3z^2 + 7z + 1}{z + 1} dz \text{ where } C \text{ is } |z| = \frac{1}{2}.$$

b. Evaluate $\int_C \frac{z-3}{z^2 + 2z + 5} dz$ where C is the circle

$|z + 1 - i| = 2$ by using Cauchy's residue theorem.

(7+7 = 14)
