

INDIAN MARITIME UNIVERSITY

(A CENTRAL UNIVERSITY, GOVT.OF INDIA)

DEC.17/JAN.18 END SEMESTER EXAMINATIONS

B.Sc. NAUTICAL SCIENCE-FIRST SEMESTER

APPLIED MATHEMATICS-I(UG21T2104)

Time:3Hrs.

Max.Marks:70

Date:28.12.2017

**NOTE:** Attempt any FIVE questions out of 7. All questions carry equal marks. Use of approved type Scientific Calculator is allowed. (5x14=70 Marks)

1a) If  $x$  and  $y$  are real, solve the equation  $\frac{iy}{ix+1} - \frac{3y+4i}{3x+y} = 0$ .

b) Prove that  $\frac{(\cos 5\theta - i \sin 5\theta)^2 (\cos 7\theta + i \sin 7\theta)^{-3}}{(\cos 4\theta - i \sin 4\theta)^9 (\cos \theta + i \sin \theta)^5} = 1$ . (7+7 marks)

2 a) Find the roots common to the equations  $x^4 + 1 = 0$  and  $x^6 - i = 0$ .

b) Expand  $\sin^7 \theta \cdot \cos^3 \theta$  in a series of sines of multiples of  $\theta$ . (7+7 marks)

3 a) Separate  $\tan^{-1}(x + iy)$  into real and imaginary parts.

b) Prove that  $\log \left[ \frac{a+ib}{a-ib} \right] = 2i \tan^{-1} \left( \frac{b}{a} \right)$ . Hence evaluate  $\cos \left[ i \log \left( \frac{a+ib}{a-ib} \right) \right]$ . (7+7 marks)

4 a) Find the  $n^{\text{th}}$  derivative of  $\sinh 2x \cdot \sin 4x$

b) If  $y = \cos(m \log x)$  prove that  $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (m^2 + n^2)y_n = 0$ . (7+7 marks)

5 a) Using Maclaurin's series expand  $\log \sec x$ .

b) If  $f(x)$  and  $g(x)$  are respectively  $e^x$  and  $e^{-x}$ , prove that 'c' of Cauchy's Mean Value theorem is the arithmetic mean between  $a$  and  $b$ . (7+7 marks)

6a) If  $u = \sin^{-1} \left[ \frac{x+y}{\sqrt{x} + \sqrt{y}} \right]$ ,

prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$  and  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cdot \cos 2u}{4 \cos^3 u}$ .

b) If  $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ , prove that  $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z} = 0$ . (7+7 marks)

7 a) Find the maximum and minimum values of  $xy + \frac{a^3}{x} + \frac{a^3}{y}$ .

b) Find the possible percentage error in computing the resistance 'r' from the formula  $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ , if  $r_1, r_2$  are both in error by 2%. (7+7 marks)