

01/12/18

**INDIAN MARITIME UNIVERSITY**  
(A Central University, Government of India)  
**END SEMESTER EXAMINATIONS –DECEMBER 2018**

**B.Sc (Nautical Science)**  
Semester-I  
**Nautical Mathematics**  
**(UG21T4102)**

Date: 29.12.2018  
Time: 3Hrs

Maximum Marks: 70  
Pass Marks: 35

**Note: Question No. 1 is compulsory.**

Answer any 6 questions from remaining 8 questions (each of 10 marks).  
Scientific Calculator is permitted if required.

1. a. Find the  $n^{\text{th}}$  derivative of  $e^x(2x + 3)^2$   
b. Verify Rollis theorem,  $f(x) = (x + 2)^3(x - 3)^4$  and  $(-2,3)$   
c. Write the formula connecting beta and gamma functions.  
d. Find the locus of  $z$  if  $\frac{z+i}{z+2}$  is real.  
e. What is the supplementary property. *Answer*
- (5 x 2 =10 marks)

2. a. Change the order of integration of the following integral and integrate

$$\int_0^1 \int_x^{\sqrt{x}} xy \, dy \, dx$$

- b. Find by double integration the area lying between the parabola  $y = 4x - x^2$  and the line  $y = x$ .

(5 + 5 marks)

3. a. Prove

$$\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$$

- b. Find the volume of the tetrahedron bounded by the Co-ordinate planes and the plane  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

(5 + 5 marks)

4. a. In spherical triangle  $DEF$ , angle  $D = 100^\circ 35'$  sides  $e = 102^\circ 48'$  and  $f = 94^\circ 34'$ . Calculate angle  $E$ .
- b. In spherical triangle  $DEF$  side  $d = 112^\circ 3'$  angle  $E = 113^\circ 23'$  and angle  $F = 90^\circ$ . Calculate angle  $D$  and sides  $e$  and  $f$ .
- (5 + 5 marks)

5. In spherical triangle  $ABC$  angle  $C = 81^\circ 24.3'$  and sides  $c = 87^\circ 10'$  and  $a = 62^\circ 37'$ . Calculate angles  $A, B$  and side  $b$  by four part formula.
- (10 marks)

6. a. An equilateral triangle constructed in the complex plane has its one vertex at the point  $(1 + i\sqrt{3})$ . Find the complex numbers representing the other 2 vertices, origin being its circumcentre.

b. Prove that  $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} \cos \frac{n\pi}{4}$

(5 + 5 marks)

7. a. Expand  $\cos^7 \theta$  in a series of cosine multiples of  $\theta$ .
- b. If  $\tan(A + iB) = x + iy$  prove that  $x^2 + y^2 + 2x \cot 2A = 1$

(5 + 5 marks)

8. a. If  $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$  prove that

$$(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

b. If  $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$  prove that  $x \frac{\partial u}{\partial y} + y \frac{\partial u}{\partial x} = \frac{1}{2} \tan u$

(5 + 5 marks)

9. a. If  $u = F(x - y, y - z, z - x)$  prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

b. Find the minimum value of  $x^2 + y^2 + z^2$

given that  $ax + by + cz = p$ .

(5 + 5 marks)

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