

# INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

Time Bound Assignment

B.Sc(Nautical Science)-Arrear Examinations

September/October 2020

Semester – I

Nautical Mathematics

(UG11T3102)

Date: 22-09-2020

Maximum Marks: 70

Time: 3 Hrs

Pass Marks: 35

- Note:**
1. Question No. 1 is compulsory.
  2. Answer any 6 questions from remaining 8 questions.
  3. Scientific Calculator is permitted if required.

Q.1

(2 x 5 = 10 marks)

- (a) Find the  $n^{\text{th}}$  derivative of  $\frac{x}{(x+9)(x+5)}$ .
- (b) Evaluate  $\int_0^1 \int_0^y x^2 y \, dx dy$
- (c) Simplify  $\frac{(\cos 5\theta - i \sin 5\theta)^2 (\cos 7\theta + i \sin 7\theta)^{-3}}{(\cos 4\theta - i \sin 4\theta)^9 (\cos \theta + i \sin \theta)^5}$
- (d) Evaluate  $\lim_{x \rightarrow 0} \left\{ \frac{1}{x} - \frac{1}{\sin x} \right\}$
- (e) Verify Lagrange's Mean value theorem for  $f(x) = (x-1)(x-2)(x-3)$  in  $[0,4]$ .

Q.2

- (a) Change to polar coordinates and hence evaluate  $\int_0^a \int_y^a \frac{x}{\sqrt{x^2 + y^2}} \, dx dy$

(5 marks)

- (b) Evaluate  $\int_0^1 \int_0^{1-x} \int_0^{x-y} \frac{dx dy dz}{(1+x+y+z)^3}$

(5 marks)

Q.3

- (a) Evaluate the volume bounded by cylinder  $x^2 + y^2 = 4$  and the planes  $y + z = 4, z = 0$ .

(5 marks)

- (b) Change the order of integration and hence evaluate the integral

$$\int_0^1 \int_x^{2-x} \frac{x}{y} \, dx dy$$

(5 marks)

Q.4 (a) In spherical triangle PZX, right angled at Z,  $p = 110^{\circ} 20'$  and  $z = 84^{\circ} 12'$ . Find the value of P. **(5 marks)**

(b) In spherical triangle ABC,  $a = 49^{\circ} 08'$ ,  $b = 58^{\circ} 23.0'$ ,  $C = 71^{\circ} 20'$ . Find c . **(5 marks)**

Q.5 (a) In a spherical triangle CDE, calculate the angle C, D, if  $c = 87^{\circ} 10'$ ,  $d = 62^{\circ} 37'$  and  $e = 100^{\circ} 10'$ . **(5 marks)**

(b) In spherical triangle PQR,  $P = 57^{\circ} 30.5'$ ,  $Q = 95^{\circ} 17'$ ,  $R = 70^{\circ} 11'$ . Calculate p. **(5 marks)**

Q.6 (a) If  $y = (x^2 - 1)^n$ , prove that  $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$  **(5 marks)**

(b) Verify whether  $(-1, 0)$  is a stationary value of  $z = x^3 + 3xy^2 - 3x$  **(5 marks)**

Q.7 (a) Find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  if  $u = \tan^{-1} \left( \frac{x^3 + y^3}{x + y} \right)$  **(5 marks)**

(b) If  $z = x + y$  and if  $x = e^u + e^{-v}$  and  $y = e^{-u} - e^v$  then find the value of  $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$  **(5 marks)**

Q.8 (a) If  $\alpha$  and  $\beta$  are roots of equations  $x^2 - 2x + 4 = 0$  then show that

$$\alpha^n + \beta^n = 2^{n+1} \cdot \cos\left(\frac{n\pi}{3}\right) \quad \textbf{(5 marks)}$$

(b) Find the values of  $(1 - i\sqrt{3})^{1/4}$  **(5 marks)**

Q.9 (a) Separate into real and imaginary parts  $\sqrt{i}^{\sqrt{i}}$  considering the principal value. **(5 marks)**

(b) Express  $\sin^5 \theta$  in terms of multiples of  $\theta$ . **(5 marks)**