

**Indian Maritime University**  
**(A Central University, Govt of India)**  
**End Semester Examinations – December 2023**

**Programme Name: B.Sc Nautical Science**

**Semester: I**

**Subject Code: UG21T5102**

**Subject Name: Mathematics**

Date: 20.12.2023

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.
- (iii) Use of scientific calculator is permitted.

Section A

**Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.**

1. A right spherical triangle has an angle  $C=90^\circ$ ,  $a=50^\circ$  and  $c=80^\circ$ . find the angle B
  - a)  $45.33^\circ$
  - b)  $78.66^\circ$
  - c)  $77.87^\circ$
  - d)  $75.89^\circ$
2. When the plane passes through the centre of the sphere, the circle is defined as .....
  - a) small circle
  - b) great circle
  - c) hemisphere
  - d) diameter
3. The first, second and third derivatives of a cubic polynomial  $f(x)$  at  $x=1$  are  $1^3$ ,  $2^3$  and  $3^3$  respectively. Then the value of  $f(0)+f(1)-2f(-1)$  is ?
  - a) 76
  - b) 86
  - c) 126
  - d) 41.5
- 4) The degree of the homogeneous function  $w = \frac{x+2y+3z}{x^8+y^8+z^8}$  is,
  - a) 7
  - b) -7
  - c) 8
  - d) -8

5) If  $u = x^3 + y^3$  then  $\frac{\partial^2 u}{\partial x \partial y}$  is equal to,

- (a) -3 (b) 3  
(c) 0 (d)  $3x+3y$

6. A vector field which has a vanishing divergence is called a .....

- a) solenoidal field b) rotational field  
c) hemispheroidal field d) irrotational field

7. what is the value of  $\Gamma(1/2)$ ?

- a)  $\sqrt{\pi}$  b)  $\sqrt{\pi}/\sqrt{2}$  c)  $\sqrt{\pi}/2$  d)  $\pi/2$

8) If  $\beta(n, 3) = \frac{1}{3}$  and n is a positive integer, then n=

- (a) 1 (b) 2  
(c) 3 (d) 4

9. Find the eigen vector for value of  $\lambda = -2$  for the given matrix  $A = \begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$

- a)  $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$  b)  $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$  c)  $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$  d)  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

10. If A is a  $3 \times 5$  matrix then the rank of  $A^T$  is at most .....

- a) 4 b) 2 c) 5 d) 3

### Section B

#### **Five Questions of 02 Marks each**

11. In a spherical triangle ABC, angle C =  $90^\circ$  angle B =  $30^\circ$  side AB =  $70^\circ$  find side AC

12. State the supplementary theorem of polar triangle.

13. Show that the area between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$  is  $\frac{16}{3}a^2$

14. If W is the set of  $2 \times 2$  matrices of the form  $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ , show that W is a subspace of space V of all  $2 \times 2$  matrices.

15. Prove that  $\nabla r^n = nr^{n-2}\vec{r}$   $\vec{r} = xi+yj+zk$

### Section C

**7 Questions of 10 Marks each of which any 05 questions to be answered.**

16. (i) In spherical triangle PZX, side  $z=70^{\circ}45'$  side  $x=62^{\circ}10'$  and side  $p=46^{\circ}19'$   
find angles P,Z,X (5Marks)

(ii) In spherical triangle ABC, angles  $A=81^{\circ}24.3'$  , $B=61^{\circ}31.7'$  , $C=102^{\circ}58'$   
calculate sides a,b,c (5Marks)

17.(i) In spherical triangle ABC if  $a=49^{\circ}8'$  , $b=58^{\circ}23'$  and  $c=71^{\circ}20'$  find A and B  
using four part formula (5Marks)

(ii) In spherical triangle NBC,  $N=40^{\circ}44'$  , $n=36^{\circ}13'$  and  $C=90^{\circ}$  find b and B using  
Napier's rule (5Marks)

18a) Prove that  $\beta(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$  . Hence evaluate  $\int_0^{\infty} \frac{x^{10}-x^{18}}{(1+x)^{30}} dx$ .  
(5Marks)

18b) Evaluate  $\int_0^1 \int_{y^2}^1 \int_0^{1-x} (x) dz dx dy$   
(5Marks)

19.(i) If  $\vec{r}=xi+yj+zk$  show that (a)  $\nabla \cdot \vec{r}=3$  (b)  $\nabla \times \vec{r} = 0$  (5Marks)

(ii) Show that  $\text{div}(\text{grad } r^n) = \nabla^2 r^n = n(n+1)r^{n-2}$  (5Marks)

20. (i) Express  $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d \theta$  in terms of gamma function (5Marks)

(ii) Prove that  $\beta(m, \frac{1}{2}) = 2^{2m-1} \beta(m, m)$  (5Marks)

21a) Find the eigen values and eigen vectors of the matrix  $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ .  
(6Marks)

21b) Evaluate  $\int_0^{\infty} x^{1/4} e^{-\sqrt{x}} dx$  (4Marks)

22. Evaluate the following integral by changing the order of integration :

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dx dy .$$

(10 Marks)

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