Indian Maritime University

(A Central University, Govt of India)

End Semester Examinations – December 2024

Programme Name: B Tech (ME)

Semester: First

Subject Code: UG11T5102

Subject Name: Engineering Mathematics 1

Date: 12.12.2024	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

Section A **(10X1=10 Marks)**

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

1. The system of equations represented by a matrix AX = B is inconsistent if

   A. R(A) is not equal to R(A:B)

   B. R(A) = R(A:B)

   C. R(A) < R(A:B)

   D. R(A) > R(A:B)

2. Consider the following matrix A. Which is the correct statement.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   A. The matrix A is not a diagonal matrix.
   B. The eigen values of matrix are 1, 2 and 4
   C. The eigen values of matrix are 1 and 2
   D. None of the other options

3.Let X and y be two arbitrary, 3x3 nin -zero, skew -symmetric matrices and Z be an arbitrary 3x3 non-zero symmetric matrix, Then which of the following matrices is skew symmetric?

a) $y^3z^4 - z^4y^3$ b) $x^{44} + y^{44}$ c) $x^4z^3 - z^4x^3$ d) None of these

4) The curve passing through (0,1) and satisfying sin $(\frac{dy}{dx})$ =c′ is

a) cos $\{\frac{(y-1)}{x}\}$=c′ b) sin $\{\frac{(y-1)}{x}\}$=c′ c) cos $\{\frac{x}{(y-1)}\}$=c′

d) sin $\{\frac{x}{(y-1)}\}$=c′

5) The complementary function of $(D^3+D^2-D-1)y=0$ is

a) $c_1e^x-(c_2+c_3x)e^{-x}$ b) $c_1e^{-x}+(c_2+c_3x)e^x$ c) $c_1e^x+(c_2+c_3x)e^{-x}$

d) $c_1e^{-x}+(c_2+c_3x)e^x$

6) **The number of arbitrary constants in the particular solution of a differential equation of third order is:**

a) **3** b) **2** c) **1** d) **0**

7)The number of non-zero rows in an echelon form is called?

a) rank of a matrix b) cofactor of the matrix c) reduced echelon form
d) conjugate of the matrix

8) The curve x^3+y^3 =3axy has symmetry about

A. x-axis
B. y-axis
C. about line y=x
D. about y=-x

9) The value of the line integral $\int_c (2xy^2dx+2x^2ydy+dz)$ along a path joining the origin and the point (1,1,1) is

a) 0 b) 2 c) 4 d) 6

10) . $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)dxdy=$

a) $\frac{3}{35}$ b) $\frac{7}{60}$ c) $\frac{4}{49}$ d) $\frac{2}{15}$

Section B

Five Questions of 02 Marks each (5X2=10 Marks)

11. Find nth order derivative of $y=e^{3x}\cos^2x$

12. Find rank of the given matrix by reducing to echelon form A= $\begin{bmatrix}0&1&2&1\\1&2&3&2\\3&1&1&3\end{bmatrix}$

13. Solve $\frac{dy}{dx} = e^{2x-3y} + 4x^2e^{-3y}$

14. A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 3$, where t is the time. Find the component of velocity and acceleration at t=1 in the direction $\vec{I} + \vec{J} + 3\vec{K}$

15. If u=xy-yz-zx, v=$x^2+y^2+z^2$, $w = x + y - z$, determine whether they are functionally related or not, if so find the relationship between them.

Section C

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16.a) If $y=(x^2-1)^n$, then show that

$$(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$ (5 marks)

16. b) Using Lagranges's method of undetermined multipliers find the maximum and minimum distances of the point (3,4,12) from the sphere

$$x^2 + y^2 + z^2 = 4.$$ (5 marks)

17.a) If $u = sin^{-1}\sqrt{\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}}$; prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{tan^2 u}{12}\right)$$ (5 marks)

17.b) Find the directional derivative of f(x,y,z)=$4e^{2x-y+z}$ at the point

(1,1,-1) in the direction towards the point (-3,5,6) (5 marks)

18.a) The upward speed $v(t)$ of a rocket at time t is approximated by

v(t) = at^2 + bt + c, $0 \le t \le 100$ where a, b, and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second. Find the speed at time t = 15 seconds. Solve the problem using Gaussian Elimination. (6 marks)

18.b) Find the eigenvalues and eigenvectors of A=$\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ (4 marks)

19. a) Find the characteristics equation of the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} . Also, find the matrix represented by

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ (5 marks)

(b) Test for consistency

$x + y + z = 6$; $x + 2y + 3z = 14$; $x + 4y + 7z = 30$ and solve them.

(05)

20.a) Define Hermitian Matrix and Skew Hermitian Matrix.

If A= $\begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$, show that AA* is Hermitian matrix,

where A* is the conjugate transpose of A.

(04)

b) Find the Eigen values and Eigen vectors for the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

(06)

21.a) If c is a simple closed curve in the xy-plane not enclosing the origin, show that using Green's theorem $\int_c F.dR = 0$, where F= $\frac{y\,I - x\,J}{X^2+Y^2}$ (5 marks)

21.b) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

(05)

22.a) Prove that $\bar{F} = (6xy + z^3)\bar{\imath} + (3x^2 - z)\bar{\jmath} + (3xz^2 - y)\bar{k}$ is irrational. Find a scalar function $\emptyset$ (x, y, z)such that $\bar{F} = \nabla \emptyset$.

(05)

b) Find the work done in moving the particle under the field of force given by $\bar{F} = (x^2 - y^2 + x)i - (2xy + y)j$ in the xy plane from the point (0, 0) to (1, 1) along the curve $y^2 = x$.

(05)