INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

December 2017 End Semester Examinations
B.Sc. (Nautical Science)- First Semester

Nautical Mathematics (UG21T3102)

Date: 07/12/2017 **Maximum Marks:** 70
Time: 3Hrs **Pass Marks:** 35

Note: i. Use of approved type of scientific calculator is permitted.
ii. The symbols have their usual meanings.

Part - A **Marks 5x2=10**

(All Questions are Compulsory)

Q.1 (a) Evaluate $\int_0^1 \int_0^y xy\, e^{-x^2}\, dxdy$

(b) Show that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m,m)$

(c) Simplify $\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5}$

(d) Evaluate $\lim_{x\to 0} \left\{\frac{1}{x^2} - \cot x\right\}$

(e) Verify Lagrange's Mean value theorem for $f(x) = x(x-1)(x-2)$ in [0,1/2]

Part - B **Marks 5x12=60**

(Answer any 5 of the following)

Q.2 (a) Separate into real and imaginary parts $\sqrt{i}^{\sqrt{i}}$ considering the principal value.

(b) If α, β are the roots of equation $x^2 - 2x + 4 = 0$, prove that $\alpha^n + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$ where n is an integer. (6+6)

Q.3 (a) Expand $\cos^5\theta$ in terms of sines and cosines of multiples of θ by using De-Moivre's theorem.

(b) Prove that $\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ (6+6)

Q.4 (a) Change the order of integration and evaluate

$$\int_0^a \int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}}\,dxdy$$

(b) Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2}\,dxdy$ over the annular region R between the circles $x^2+y^2=4, x^2+y^2=1$. (6+6)

Q.5 (a) Prove that $\lceil m \left| m+\frac{1}{2} = \frac{\sqrt{\pi}}{2^{2m-1}} \right| 2m$

(b) Evaluate the volume bounded by cylinder $x^2+y^2=4$ and the planes y + z = 4 , z = 0. (6+6)

Q.6 (a) In the spherical triangle PQR, Q=90^0 , PR=$84^0 12'$ and QR=$110^0 20'$. Calculate PQ.

(b) In the spherical triangle LMN, LM=$54^0 56'$, MN=$66^0 41'$ and LN=$104^0 10'$. Calculate L. (6+6)

Q.7 (a) In a spherical triangle ABC, given side c= 100^0 09', angle A = 88^0 24.5' and angle B = 97^0 46'. Calculate the third angle C.

(b) In a quadrantal triangle XYZ, side y= $78^0 14'$, side z= 49^0 08' and side x= 90^0 . Determine the angles X and Y. (6+6)

Q.8 (a) Find the value of $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2}$ if $z = \log\left(x^3+y^3-x^2y-xy^2\right)$

(b) If $y = \sin^{-1} x$, then prove that $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$ (6+6)

Q.9 (a) Find the possible percentage error in computing parallel resistance r of the two resistances r_1, r_2 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ if r_1, r_2 are each in error by plus 3%.

(b) Using Taylor's series, obtain $\tan^{-1}(1.003)$ to 4 decimal places, where $\pi = 3.1416$ (6+6)

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