

Indian Maritime University

(A Central University, Govt of India)

May-June 2018 End Semester Examinations

B. Tech (Marine Engineering)

Semester-II

Mathematics II(UG11T2202 /T1202)

Date: 12.06.2018

Max Marks:100 Marks

Time: 3 Hrs

Pass Marks: 50 Marks

PART-A

Marks: 10 x 3 =30

(All Questions are compulsory)

1. a. Find a_0 in the Fourier series for the function $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$
- b. Find the Laplace transform of $f(t) = 4$ for $0 \leq t \leq 1$
 $= 3$ for $t > 1$
- c. Find the Laplace transform of the function
 $f(x) = |t - 1| + |t + 1|, t \geq 0$
- d. Find the Inverse Laplace transform of $\frac{s^2 - 3s + 4}{s^3}$
- e. Solve $ydx - x dy + \log x dx = 0$
- f. Find the integration factor of the differential equation
 $\frac{dx}{dy} + \frac{3x}{y} = \frac{1}{y^2}$
- g. Find the Particular Integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$
- h. A speaks the truth in 75% cases and B in 80% cases. In what percentage of cases are they likely to contradict each other in stating the same fact.

i. A variate X has the probability distributions

x	:	-3	6	9
P ($X=x$)	:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(2x + 1)^2$

j. Determine the binomial distribution for which mean = 2
(variance) and mean + variance = 3.

PART-B

Marks: 5 x 14 = 70

(Answer any 5 of the following 7 questions)

2. a. Obtain the Fourier series for the functions

$$f(x) = \pi x \quad 0 \leq x < 1$$

$$= \pi(2 - x) \quad 1 \leq x < 2$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

b. Expand $f(x) = \frac{1}{4} - x$ for $0 < x < \frac{1}{2}$

$$= x - \frac{3}{4} \quad \text{for } \frac{1}{2} < x < 1$$

as the Fourier Series of Sine terms

(6 + 8 marks)

3. a. Evaluate

$$\int_{t=0}^{\infty} \int_{u=0}^t \frac{e^{-t} \sin u}{u} du dt$$

b. Given

$$L(\sin \sqrt{t}) = \frac{\sqrt{\pi}}{2S^{3/2}} e^{-\frac{1}{4S}}$$

prove that

$$L\left(\frac{\cos \sqrt{t}}{\sqrt{t}}\right) = \sqrt{\frac{\pi}{s}} e^{\frac{-1}{4\pi}}$$

(7 + 7 marks)

4. a. Solve $x^4 \frac{dy}{dx} + x^3 y + \operatorname{cosec}(xy) = 0$

- b. Find the curve for which the tangent at any point p on it bisects the angle between the ordinate at P and the line joining P to the origin.

(6 + 8 marks)

5. a. Solve $(D - 2)^2 = 8(e^{2x} + \sin 2x + x^2)$

- b. Solve by the method of variation of parameters

$$y'' - 2y' + y = e^x \log x$$

(7 + 7 marks)

6. a. The contents of 3 urns are
1 white, 2 red, 3 green balls
2 white, 1 red, 1 green balls
4 white, 5 red, 3 green balls

Two balls are drawn from an urn chosen at random. There are found to be one white and one green. Find the probability that the balls so drawn came from the third urn.

- b. A function is defined as follows.

$$\begin{aligned} f(x) &= 0 & x < 2 \\ &= \frac{1}{18}(2x + 3) & 2 \leq x \leq 4 \\ &= 0 & x > 4 \end{aligned}$$

Show that it is a density function. Find the probability that a variate having this density will fall in the interval $2 \leq x \leq 3$

(7 + 7 marks)

7. a. Find the Fourier Series for

$$\begin{aligned} f(x) &= -1 & -\pi \leq x < -\pi/2 \\ &= 0 & -\pi/2 \leq x < \pi/2 \\ &= 1 & \pi/2 \leq x < \pi \end{aligned}$$

- b. Solve $2ydx + x(2 \log x - y)dy = 0$

(7 + 7 marks)

8. a. Use Laplace transform method to solve

$$(D^3 - 3D^2 + 3D - 1)y = t^2 e^t \text{ given}$$

$$y(0) = 1; y'(0) = 0; y''(0) = -2$$

- b. X is a Poisson variable and it is found that the probability that $X = 2$ is two thirds of the probability that $X = 1$. Find the probability that $X = 0$ and the probability that $X = 3$. What is the probability that X exceeds 3.

(7 + 7 marks)
