INDIAN MARITIME UNIVERSITY

(A Central University Government of India)

END SEMESTER EXAMINATION DECEMBER 2017

Programme	**:** B.Sc (Nautical Science)	**Semester**	**:** II
Subject Name	**:** Applied Mathematics	**Subject Code**	**:** UG21T3201
Date	**:** 09.12.2017	**Maximum Marks**	**:** 70
Time	**:** 3 Hrs.	**Pass Marks**	**:** 35

Answer any 7 Questions.
All questions carry equal marks. (7x10=70 Marks)

1. a. If $u = x + y + z,\ v = x^2 + y^2 + z^2,\ w = yz + zx + xy$ prove that $grad\ u, grad\ v\ grad\ w$ are coplanar.

   b. Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(1, -2, 1)$ in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.

   c. Apply Green's theorem to evaluate $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where c is the boundary of the area enclosed by the x axis and the upper half of the circle $x^2 + y^2 = a^2$

   (3+3+4 marks)

2. a. Evaluate $div\ \bar{F}$ and $curl\ \bar{F}$ at the point $(1, 2, 3)$ given $\bar{F} = x^2yz\hat{\imath} + xy^2z\hat{\jmath} + xyz^2\hat{k}$.

   b. Apply stoke's theorem to evaluate $\int_c y\,dx + z\,dy + x\,dz$ where c is the curve of intersection of $x^2 + y^2 + z^2 = a$ and $x + z = a$.

   c. Evaluate $\int F \cdot ds$ where $F = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region $x^2 + y^2 = 4, z = 0,\ z = 3$.

   (2+4+4 marks)

3. a. Solve $y(2xy + e^x)dx = e^x dy$

b. Solve $(D-2)^2 = 8\,(e^{2x} + \sin 2x + x^2)$

c. Solve $(D^2 - 6D + 25)y = e^{2x} + \sin x + x$

(4+3+3 marks)

4. a. Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + 4y = \tan 2x$$

b. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx}\, y = \sin[2\log(1+x)]$

(5+5 marks)

5. a. Solve by Charpit's method $z = p^2x + q^2\, y$

b. Solve $(D^2 + 2DD' + D'^2)z = x^2 + xy + y^2$

where $D = \frac{\partial}{\partial x}$ $D' = \frac{\partial}{\partial y}$

(5+5 marks)

6. a. From the following table estimate the number of students who obtained marks between 40 and 45.

Marks	30-40	40-50	50-60	60-70	70-80
No of students	31	42	51	35	31

b. Find an approximate value of $\sqrt{28}$ correct to 4 decimal places.

(5+5 marks)

7. a. A reservoir discharging water through sluices at a depth Hbelow the water surface has a surface area A for various values of H as given below.

H ft	10	11	12	13	14
A sq.ft	950	1070	1200	1350	1530

If t denotes the time in minutes, the rate of fall fo the surface is given by $\frac{dH}{dt} = -\frac{48\sqrt{H}}{A}$.

Estimate the time taken for the water level to fall from 14 to 10 ft above the sluices

b. Using Gauss Seidal iteration method solve

20x + y – 2z = 17

3x + 20 y – z = -18

2x – 3y + 20z = 25

(5+5 marks)

8. a. $$\text{Evaluate } \int_0^{\infty} \frac{e^{-at} - e^{-bt}}{t}\, dt$$

b. Find Laplace inverse of $\frac{1}{2}\log\left(\frac{S^2+b^2}{S^2+a^2}\right)$

c. Using convolution theorem evaluate Laplace Inverse of $\frac{1}{(S+a)(S+b)}$

(2+4+4 marks)

9. a. $$\text{Evaluate } L\int_0^t \int_0^t \int_0^t (t\sin t)\, dt\, dt\, dt$$

b. Solve by the method of Laplace transform, the equation $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$.

(4+6 marks)

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