Indian Maritime University

(A Central University, Govt of India)

End Semester Examinations – June 2024

Programme Name: B Tech (ME)

Semester: Second

Subject Code: UG11T4201

Subject Name: Mathematics II

Date: 27.05.2024 Max Marks: 70

Duration: 03 Hrs Pass Marks: 35

General Instructions

(i) All Sections (A, B & C) are to be attempted.
(ii) Options, if any, are specified in respective section.

Section A

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

1. The curve passing through (0,1) and satisfying sin $(\frac{dy}{dx})$ =c′ is
   a) cos $\{\frac{(y-1)}{x}\}$=c′
   b) sin $\{\frac{(y-1)}{x}\}$=c′
   c) cos $\{\frac{x}{(y-1)}\}$=c′
   d) sin $\{\frac{x}{(y-1)}\}$=c′

2. The complementary function of $(D^3 + D^2 - D - 1)y = 0$ is
   a) $c_1e^x - (c_2 + c_3x)e^{-x}$
   b) $c_1e^{-x} + (c_2 + c_3x)e^x$
   c) $c_1e^x + (c_2 + c_3x)e^{-x}$
   d) $c_1e^{-x} + (c_2 + c_3x)e^x$

3. The number of arbitrary constants in the particular solution of a differential equation of third order is:
   a) 3
   b) 2
   c) 1

d) 0

4. The differential equation $(x + x^8 + ay^2)dx + (y^8 - y + bxy)dy$ is exact if

a) a = b b) a = 2 , b = 3 c) b = 2a d) b = a/ 2

5. The inverse Laplace Transform of $\frac{1}{s^2+9}$ is

a) $\frac{1}{3} sin3t$ b) $\frac{1}{3} cos3t$ c) $sin^2 3t$ d) $sin3t$

6. If L {f(t)}=$\bar{f}(s)$, then $L^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}$ is equal to

a) $\frac{1}{2}$(sin t −t cost)
b) $\frac{1}{2}$(sin t +t cost)
c) $\frac{1}{2}$(sin t −cost)
d) $\frac{1}{2}$(sin t +cost)

7. The partial differential equation of the given function z=$ax + a^2y^2 + b$ by eliminating the arbitrary constant is

a) q=$2p^2y$
b) p=$2q^2y$
c) z=px +qy +pq
d) $pq = 4xyz$

8. The value of b_1 of the Fourier series expansion for the function f(x)=x sin x, 0<x<2π

a) 2π
b) 3π
c) $-\pi$
d) π

9. The value of $\oint_c \frac{z-3}{z^2+2z+5} dz$, where c is the circle $|z| = 1$ is

a) 1
b) 0
c) 3
d) 2

10. For the function $\frac{\sin z}{z^3}$ of a complex variable the point z=0 is

a) a pole of order 4

b) a pole of order 3
c) a pole of order 2
d) not a singularity

Section B

Five Questions of 02 Marks each

11. Find a_0 of the Fourier series expansion for the following function
f(x)=$|x|, -\Pi < x < \Pi$

12. Find L$\{\frac{\sin at}{t}\}$, given that L$\{\frac{\sin t}{t}\} = tan^{-1}\{\frac{1}{s}\}$

13. Find r, if x^r is an integrating factor of the equation
$(x+y^3)dx + 6xy^2dy = 0$

14. Solve : $(x+1)\frac{dy}{dx} + 1 = 2\,e^{-y}$

15. Form the partial differential equation by eliminating the function f from the relation $z = y^2 + 2f\,(\frac{1}{x} + \log y)$.

Section C

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16 a) Find the Fourier series for
f(x)=x for $0 \le x \le 1$
= 1-x for $1 \le x \le 2$ (5 marks)

16 b) Find the Fourier series of x^2 in (-π,π). Use Parseval's identity
Prove that $\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots \ldots \ldots \ldots$ (5 marks)

17 a) Solve $x^4\frac{dy}{dx} + x^3y + cosec(xy) = 0$ (5 marks)

17 b) Solve $(D^4 + 2D^2 + 1)y = x^2\cos x$, where D≡$\frac{d}{dx}$ (5 marks)

18a) Solve by using the method of variation of parameters
$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$ (5 marks)

18 b) Solve $(1+x)^2\frac{d^2y}{dx^2}$ + (1+x) $\frac{dy}{dx}$ + y=4Cos log (1+x) (5 marks)

19a) Find the particular integral of $y = \frac{1}{D^2-5D+16} x^2 e^{3x}$ (4 marks)

19b) Solve by separation of variables $u_{xx} - u_y = 0$ (6 marks)

20a) Find $L^{-1}\{\frac{1}{2}\log \frac{s^2+b^2}{s^2+a^2}\}$ (5 marks)

20b) Use unit step function to evaluate (5 marks)

$$f(t)=\begin{cases} t, & 0 < t < 2 \\ 2, & t > 2 \end{cases}$$

21a) Using convolution theorem to find $L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}$ (5 marks)

21b) Solve the equation using Laplace Transform

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t, \quad x(0) = 0, \; x'(0) = 1$$ (5 marks)

22 a) Determine the analytic function whose real part is $(y + e^x cosy)$ (5 marks)

b) Evaluate : $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, *where C is the circle* $|z| = 2$ (5 marks)