

Indian Maritime University
(A Central University, Govt of India)

Supplementary Examinations – March/April 2024

Programme Name: B Sc (NS)

Semester: I

Subject Code: UG21T5102

Subject Name: Mathematics

Date: 12.04.2024

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.
- (iii) Scientific Calculator is permitted.

Section A

Objective type questions/Fill in the blanks(01 Marks Each)

1. Matrix has a value. This statement

- a) is always true b) depends upon the matrices c) is false

2. The sum of the squares of the eigen values of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ is

- a) 10 b) 100 c) 38

3. If $u = x^y$ then $\frac{\partial u}{\partial y}$ is

- a) 0 b) yx^{y-1} c) $x^y \log x$

4. The sum of the three angles must lie between 180° and 540° . True or False

5. A great circle is

6. The first derivative of $e^{-x}x^3$ is.....

7. If an error of 1% is made in measuring its length and breadth the percentage error in the area of a rectangle is

- a) 0.2% b) 0.02% c) 2% d) 1%

8. The value of $\beta(2,1) + \beta(1,2)$ is

- a) 1 b) 2 c) 0

9. To change cartesian coordinates (x, y, z) to spherical polar coordinate (r, θ, ϕ) ; $dx dy dz$ is replaced by
10. If the directional derivative of $f = ax + by + cz$ at $(1, 1, 1)$ has maximum magnitude 4 in direction parallel to x axis then the values of a, b, c are
- a) $(-2, 2, 2)$ b) $(2, -2, 2)$ c) $(2, 2, -2)$

Section B

Short Answer Type Questions (02 Marks Each)

11. If $u = \sin^{-1} \left(\frac{x+2y+3z}{x^8+y^8+z^8} \right)$ find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$
12. In spherical triangle PQR, $p = 62^\circ 10.1'$, $q = 111^\circ 35.2'$, $r = 63^\circ 33'$ Calculate P.
13. If $u \ddot{F} = \nabla v$, where u, v are scalar fields and \ddot{F} is a vector field, show that $\ddot{F} \cdot \text{curl } \ddot{F} = 0$
14. Change the order of integration and hence prove $\int_0^\infty \int_x^\infty \frac{1}{y} e^{-y} dx dy = 1$
15. Determine whether the set $\{t^2+2t-3, t^2+5t, 2t^2-4\}$ of vectors is linearly independent.

Section C

Answer five out of seven questions (10 Marks Each)

16. a) Define quadrantal spherical triangles and mention any two properties of the same (5)
- b) In spherical triangle PQR, $PQ = 52^\circ 11'$, $Q = 69^\circ 47'$ and $QR = 90^\circ$ Calculate P, R and PR (5)
17. a) Define spherical triangle and its properties (5)
- b) In spherical triangle WXY, $W = 88^\circ 24.5'$, $x = 98^\circ 10'$, $y = 100^\circ 09'$ find w and X. (5)
18. a) State Leibnitz' theorem (3)
- b) If $y = (\sin^{-1} x)^2$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - x^2y_n = 0$. hence find $(y_n)_0$ (7)
19. a) If $u = \tan^{-1} \frac{x^3+y^3}{x-y}$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ (4)
- b) Find the extreme points of the function $z = x^3 + y^3 - 3axy$ (6)
20. a) Evaluate in terms of gamma function, the integral $\int_0^\infty e^{-x^2} dx$ (5)
- b) Find by double integration the areas enclosed by the curve

$y^2=4ax$ and its latus rectum

(5)

21.a) Find the directional derivative $f=(x^2 + y^2 + z^2)^{-1/2}$ at the point $P(3,1,2)$ in the direction of the vector $yi+zj+xyk$ (5)

b) If $\vec{r} = xi+yj+zk$ and $r=|\vec{r}|$ show that $\text{div}\left(\frac{\vec{r}}{r^3}\right)=0$ (5)

22. If $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ find eigen values and eigen vectors of

a) A^2-2A+I b) $4A^{-1}$

(10)



