INDIAN MARITIME UNIVERSITY

(A CENTRAL UNIVERSITY, GOVT. OF INDIA)

December 2017 End Semester Examination
B. Tech. (Marine Engineering) First Semester

Mathematics - I (Subject Code: UG11T3102/ UG11T2102 UG11T1102)

Date: 07/12/2017 **Max Marks:** 100
Time: 3 Hrs. **Pass Marks:** 50

PART – A **(3 x10 = 30)**

Compulsory Questions: (The symbols have their usual meanings.)

1. (a) Find the nth derivative of $\log(4x^2 - 1)$.

   (b) If $z = e^{ax+by} f(ax - by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.

   (c) Find the radius of curvature at any point $(at^2, 2at)$ of the parabola $y^2 = 4ax$.

   (d) If $u = xy - yz - zx, v = x^2 + y^2 + z^2$ and $w = x + y - z$, determine whether they are functionally related, if so find the relationship between them.

   (e) If $u = e^{xy}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{u}\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right\}$.

   (f) Evaluate the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar coordinates.

   (g) Show that $\int_0^\infty \frac{x^4}{4^x} dx = \frac{\Gamma(5)}{(\log 4)^5}$.

   (h) Find the directional derivative of function $f = xy^2 + yz^2$ at the point $(1, -1, 1)$ along the vector $\hat{\imath} + 2\hat{\jmath} + \hat{k}$.

   (i) Using Cayley Hamilton theorem find the A^{-1} of matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

   (j) Evaluate the integral $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = 1/2$.

PART – B **(14 x 5 = 70)**

Answer any <u>FIVE</u> of the following questions

2. (a) If $y = e^{m\cos^{-1}x}$, prove that
   (i) $(1-x^2)y_2 - xy_1 = m^2y$
   (ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. **[3+4]**

   (b) Find the asymptotes of the curve $y^3 - 2xy^2 - x^2y + 2x^3 + 3y^2 - 7xy + 2x^2 + 2y + 2x + 1 = 0$. **[7]**

3. (a) If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, prove that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$. **[7]**

   (b) Using the method of Lagrange's multipliers find the points on the surface $z^2 = xy + 1$ nearest to the origin. **[7]**

4. (a) Using the rule of differentiation under the sign of integration prove that $\int_0^{\pi/2} \frac{\log(1+y\sin^2 x)}{\sin^2 x}dx = \pi\left[\sqrt{(1+y)} - 1\right]$. **[7]**

   (b) Prove that
   (i) $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m,m)$ and hence
   (ii) $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. **[5+2]**

5. (a) Evaluate the double integral $\iint xy(x+y)dxdy$ over the area between $y = x^2$ and $y = x$. **[7]**

   (b) Evaluate triple integral $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\, dz\, dx\, dy$. **[7]**

6. (a) A particle moves on the curve $x = 2t^2,\ y = t^2 - 4t,\ z = 3t - 5$ where t is the time. Find the component of the velocity and acceleration at $t = 1$ in the direction $i - 3j + 2k$. **[6]**

   (b) Show that vector $\vec{F} = (2xy^2 + yz)\hat{\imath} + (2x^2y + xz + 2yz^2)\hat{\jmath} + (2y^2z + xy)\hat{k}$ is irrotational. Find a scalar function $\emptyset$ such that $\vec{F} = \nabla\emptyset$ **[8]**

7. (a) Discuss the consistency of the following system of equations and solve it if consistent.
   $x + 2y - z = 3,$
   $3x - y + 2z = 1,$
   $2x - 2y + 3z = 2,$
   $x - y + z = -1$ **[7]**

   (b) Let $F(z) = u(x,y) + iv(x,y)$ be an analytic function of z. If $u = x^4 - 6x^2y^2 + y^4$ then find the v and express $f(z)$ in terms of z. **[7]**

8. (a) Find the curve on which functional $\int_0^2 (x + y')y'dx$ with $y(0) = 0$ and $y(2) = 1$ can be extremized. **[7]**

(b) Using simplex method solve the following LPP

Maximize $Z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \leq 2$, $5x_1 + 2x_2 \leq 10$, $3x_1 + 8x_2 \leq 12$, $x_1,\ x_2 \geq 0$. **[7]**

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