

INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)
END SEMESTER EXAMINATION-DECEMBER 2019
B.Sc(Nautical Science)
Semester – I
Applied Mathematics - I
(UG21T2104)

Date: 09.12.2019

Max Marks: 70

Time: 3 Hrs

Pass Marks : 35

7 questions out of 9. All questions carry equal marks.

1. a. If x and y are real, solve the equation $\frac{iy}{ix+1} - \frac{3y+4i}{3x+y} = 0$

b. Express the following in the form $a + ib$, $\frac{2 - i\sqrt{3}}{1+i}$

(5+5 marks)

2. Find the locus of Z given by

a. When $\frac{z+i}{z+2}$ is real;

b. When $\frac{z-1}{z-2}$ is purely imaginary.

(5+5 marks)

3. a. If α, β be the roots of $x^2 - 2x + 4 = 0$ prove that

$$\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}$$

b. Prove that if $x = \cos \alpha + i \sin \alpha$; $y = \cos \beta + i \sin \beta$.

$z = \cos r + i \sin r$ and $x + y + z = 0$ then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

(5+5 marks)

4. a. Find the n^{th} derivative of $e^x \cos x \cos 2x$

b. If $y = e^{m \cos^{-1} x}$ prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$$

(5+5 marks)

5. a. Verify Rolle's theorem if $y = e^x(\sin x - \cos x)$ in $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

b. Verify Lagrange's Mean Value theorem for
 $f(x) = (x - 1)(x - 2)(x - 3)$ in $(0, 4)$.

(5+5 marks)

6. a. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$

b. If $u = e^{xyz}$ find the value of $\frac{\partial^2 u}{\partial x \partial y \partial z}$.

(5+5 marks)

7. Find the maximum and minimum values of

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$

(10 marks)

8. a. The sum of 3 numbers is constant. Prove that their product is maximum when they are equal.

b. Divide 24 into 3 parts such that the continued product of the first square of the second, and the cube of the third may be maximum.

(5+5 marks)

9. Expand $\sin^5 \theta \cos^2 \theta$ in a series of sine multiples of θ .

(10 marks)
