

**INDIAN MARITIME UNIVERSITY**  
(A Central University Government of India)  
END SEMESTER EXAMINATIONS-June/July 2019  
**B.Sc Nautical Science**  
**Semester-I**  
**Applied Mathematics paper - I**  
**(UG21T2104)**

**Date: 01-07-2019**

**Duration: 3 hrs.**

**Maximum Marks: 70**

**Pass Marks: 28**

**Note: Attempt any FIVE questions out of seven question.**  
**All questions carry equal marks.**  
**Use of approved type Scientific Calculator is permitted.**

Q.1 a) Find the complex number  $z$  if  $\arg(z+1) = \frac{\pi}{6}$  and  $\arg(z-1) = \frac{2\pi}{3}$ .  
(7 marks)

b) Find all values of  $(1 - i\sqrt{3})^{\frac{1}{4}}$ .  
(7 marks)

Q.2 a) If  $\cosh(u+iv) = x + iy$ , prove that

(i)  $\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1$       (ii)  $\frac{x^2}{\cos^2 v} - \frac{y^2}{\sin^2 v} = 1$       (7 marks)

b) Prove that :  $\sin^5 \theta = \frac{1}{10} (\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$ ,      (7 marks)

Q.3 a) Find the  $n$ th derivative of  $e^{5x} \cos x \cos 3x$ .      (7 marks)

b) If  $y = (\sin^{-1} x)^2$ , show that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$ .  
(7 marks)

Q.4 a) Verify Rolle's theorem for  $f(x) = (x+2)^3(x-3)^4$  in  $(-2,3)$ .      (7 marks)

b) Prove that  $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \dots$       (7 marks)

Q.5 a) Evaluate the limit :  $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x^3}$ .      (7 marks)

b) If  $v = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ , prove that  $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$ . (7 marks)

Q.6 a) If  $u = \log \frac{x^4 + y^4}{x + y}$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ . (7 marks)

b) Given  $u = \sin \left( \frac{x}{y} \right)$ ,  $x = e^t$  and  $y = t^2$ , find  $\frac{du}{dt}$  as a function of  $t$ . Verify your result by direct substitution. (7 marks)

Q.7 a) Find the percentage error in the area of an ellipse if one percent error is made in measuring the major and minor axes. (7 marks)

b) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. (7 marks)

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