**INDIAN MARITIME UNIVERSITY**

(A Central University, Government of India)

END SEMESTER EXAMINATION-DECEMBER 2019

**B.Sc(Nautical Science)**

**Semester – I**

**Nautical Mathematics**

**(UG21T3102)**

| | |
|---|---|
| Date: 12.12.2019 | Max Marks: 70 |
| Time: 3 Hrs | Pass Marks : 35 |

Note: Question 1 is compulsory.

Solve any 6 questions from remaining 8 questions.

Use of approved type scientific calculator is permitted.

The symbols have their usual meaning.

---

Q.1 a) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\, dx\, dy$.

b) In a spherical triangle PQR, p = $67^0$, q = $54^0$, P = $39^0$. Find Q?

c) Simplify : $\frac{(\cos 3\theta + i \sin 3\theta)^4 (\cos 4\theta - i \sin 4\theta)^5}{(\cos 4\theta + i \sin 4\theta)^3 (\cos 5\theta + i \sin 5\theta)^{-4}}$ .

d) Evaluate the limit : $\lim_{x \to 0} \frac{x \cos x - \log(1+x)}{x^2}$ .

e) Verify Rolle's theorem for f(x) = $(x+2)^3(x-3)^4$ in (-2 , 3).

(10 marks)

Q.2 a) Evaluate $\iint \frac{r\, dr\, d\theta}{\sqrt{a^2+r^2}}$ , over one loop of the lemniscate $r^2 = a^2 \cos 2\theta$.

b) Find, by double integration, the area enclosed by the curves

$y = \frac{3x}{(x^2+2)}$ and $4y = x^2$. (5 + 5 marks)

Q.3 a) Evaluate : $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx\, dy$.

b) State and prove the relation between Beta and Gamma function.

(5 + 5 marks)

Q.4 a) In spherical triangle ABC, a = 49° 08', b = 58°23' and C = 71°20'.

Calculate A and c.

b) In spherical triangle PZX, right angled at Z, p = $110^0$ 20' and

z = $84^0$ 12'. Find the value of P, X and x.(5 + 5 marks)

Q.5 a) In spherical triangle DEF, D = 64° 36', e = 90° and E = 76°47'. Calculate d, f and F.

b) In spherical triangle PQR, Q = 74° 52.3', R = 71°20' and p = 49°08'. Calculate P and r. (5 + 5 marks)

Q.6 a) Find tanh x, if 5 sinh x – cosh x = 5.

b) Expand $\cos^8 \theta$ in a series of cosines of multiples of θ.

(5 + 5 marks)

Q.7 a) If cosh (u + iv) = x + iy , Prove that (i) $\frac{x^2}{cosh^2 u} + \frac{y^2}{sinh^2 u} = 1$.

(ii) $\frac{x^2}{cos^2 v} + \frac{y^2}{sin^2 v} = 1$.

b) Find all the values of $(\frac{1}{2} + \frac{i\sqrt{3}}{2})^{3/4}$. (5 + 5 marks)

Q.8 a) If y = $\sin^{-1}$ x, prove that $(1-x^2)\, y_{n+2} - (2n+1)\, xy_{n+1} - n^2\, y_n = 0$.

b) If $u = tan^{-1}\frac{x^3+y^3}{x+y}$ , prove that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. and

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\,\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$. (5 + 5 marks)

Q.9 a) If z = tan(y+ax) – $(y-ax)^{3/2}$ , show that $\frac{\partial^2 z}{\partial x^2} = a^2\frac{\partial^2 z}{\partial y^2}$.

b) If u = x log xy where $x^3 + y^3 + 3xy = 1$.Find $\frac{du}{dx}$. (5 + 5 marks)

*****