**Indian Maritime University**

**(A Central University, Govt of India)**

**End Semester Examinations – December 2023**

**Programme Name: B Tech (ME)**

**Semester: First**

**Subject Code: UG11T4101**

**Subject Name: Mathematics I**

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| Date: 15.12.2023 | Max Marks: 70 |
| Duration: 03 Hrs | Pass Marks: 35 |

## Section A

**Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.** **1x10=10**

1. If $y=x^x$ , then , then $y_2 =$

a) $\frac{1}{2}{y_1}^2-\frac{y}{x}$ b) $\frac{1}{y}{y_1}^2+\frac{x}{y}$ c) $\frac{x}{y}{y_1}^2+\frac{y}{x}$ d) $\frac{1}{2}{y_1}^2+\frac{y}{x}$

2.If $v=(x^2+y^2+z^2)^{\frac{-1}{2}}$ , $\frac{\delta^2 v}{\delta x^2}+\frac{\delta^2 v}{\delta y^2}+\frac{\delta^2 v}{\delta z^2}$ is

a) -1/2 b)1/2 3)0 4)1

3. If $A=\begin{bmatrix}2 & x-3 & x-2\\ 3 & -2 & -1\\ 4 & -1 & -5\end{bmatrix}$ is a symmetric matrix, then x is

a) 3 b) 6 c) 8 d) 0

4. The number of non-zero rows in an echelon form is called?

a) rank of a matrix b) cofactor of the matrix c) reduced echelon form d) conjugate of the matrix

5. The value of the line integral $\int_c (2xy^2dx+2x^2ydy+dz)$ along a path joining the Origin and the point (1,1,1) is

a) 0 b) 2 c) 4 d) 6

6. A vector field which has a vanishing divergence is called as _____
a) Solenoidal field b) Rotational field c) Hemispheroidal field d) Irrotational field

7. Find the curl of $\vec{f}(x,y,z) = x^2\vec{i} + xyz\vec{j} - z\vec{k}$ at the point (2,1,-2).
a) $2\vec{i} + 2\vec{k}$ b) $-2\vec{i} - 2\vec{j}$ c) $4\vec{i} - 4\vec{j} + 2\vec{k}$ d) $-2\vec{i} - 2\vec{k}$

8. Statement 1: The rank of $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ is 3.

Statement 2: The determinant of the matrix $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ is not equal to zero.

a) Statement 1 is true; Statement 2 is true; Statement 2 is a correct explanation of Statement 1
b) Statement 1 is true; Statement 2 is true; Statement 2 is not the correct explanation of Statement 1
c) Statement 1 is true; Statement 2 is false.
d) Statement 1 is false; Statement 2 is true

9. $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy =$

a) $\frac{7}{60}$ b) $\frac{3}{35}$ c) $\frac{4}{49}$ d) $\frac{2}{15}$

10. If x=uv and y=$\frac{u+v}{u-v}$, then $\frac{\delta(u,v)}{\delta(x,y)}$ is a function of f(u,v),then f(2,1) is
a) 0.125 b) 0.5 c) 0.2 d) 0.3

## Section B

**Five Questions of 02 Marks each (5x2=10)**

11. Find the nth derivative of $\frac{x}{2x^2+x-3}$.

12. Show that div curl F= $\nabla$. ($\nabla$X F) =0 where $\nabla$ is the vector operator del and F is the vector point function.

13. Determine if the following matrix is linearly dependent or not.
A=$\begin{pmatrix} 5 & 3 \\ 10 & 6 \end{pmatrix}$

14. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$

15. Show that $\beta\left(\frac{1}{2},\frac{1}{2}\right)=\pi$

## Section C

**07 Questions of 10 Marks each, of which any 05 questions to be answered.**

16 a) If $\cos^{-1}\left(\frac{y}{b}\right)=\log(\frac{x}{a})^n$, then prove that
$x^2y_{n+2}+(2n+1)xy_{n+1}+2n^2y_n=0$ (5)

16 b) In a plane triangle find the maximum value of Cos A Cos B Cos C using lagrange's method of undetermined multipliers. (5)

17.a) If $u=\frac{(x^2+y^2)^m}{2m(2m-1)}+x\,\Phi(\frac{y}{x})+\Psi(\frac{y}{x})$, then prove that
$x^2\frac{\partial^2U}{\partial X^2}+2xy\frac{\partial^2U}{\partial x\partial y}+y^2\frac{\partial^2U}{\partial X^2}=(x^2+y^2)^m$ (6)

17. b) If $u=f(x,y), where\ x=rcos\theta, y=rsin\theta$, then prove that
$(\frac{\partial u}{\partial x})^2+(\frac{\partial u}{\partial y})^2=(\frac{\partial u}{\partial r})^2+\frac{1}{r^2}(\frac{\partial u}{\partial \theta})^2$ (4)

18.a) Find the Eigen values and Eigen vectors of the matrix.

$$\begin{pmatrix} 2 & 1 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$$ (7)

18 b) Find the rank of the matrix using row echelon form

$$A=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}$$ (3)

19.a) Test for consistency and solve the equations 5x+3y+7z=4, 3x+26y+2z=9, 7x+2y+10z=5. (5)

19.b) If $A=\begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & 6 & 4+2i \end{bmatrix}$, show that AA* is a Hermitian Matrix where A* is the conjugate transpose of A. (5)

20.a) A vector field is given by $\vec{A}=(y+z)\vec{\imath}+(z+x)\vec{\jmath}+(x+y)\vec{k}$. Show that the field is irrotational and find the velocity potential. (5)

20b) If u=x+y+z, v=$x^2+y^2+z^2$ , w=xy+yz+zx, prove that

grad u, grad v and grad w are coplaner. (5)

21.a) a) Prove that : $\beta(m,n) = \frac{\Gamma m\ \Gamma n}{\Gamma(m+n)}$ (5)

b) Change the order of integration in $\int_0^\infty \int_y^\infty \frac{e^{-x}}{x}\, dxdy$ and hence evaluate. (5)

22.a) If the area bounded by $\frac{x}{a}+\frac{y}{b}=1, x$ axis and y axis, rotated about y axis, then find the volume of the sold generated. (5)

22.b) Using Green's theorem, evaluate $\int_c [(y-sinx)dx+cosx\, dy)]$, where c is plane triangle enclosed by the lines y=0, x=$\frac{\Pi}{2}$ & y=$\frac{2}{\Pi}x$ (5)