

**Indian Maritime University**  
**(A Central University, Govt of India)**  
**End Semester Examinations – June 2023**  
**Programme Name: B Sc (NS)**  
**Semester: I**  
**Subject Code: UG21T5102**  
**Subject Name: Mathematics**

Date: 07.06.2023

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.
- (iii) Scientific Calculator is permitted.

Section A

**Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.**

1.  $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$  is called .....

- a) cosine formula   b) sine formula   c) Both a and b   d) none of these

2. In Napier's rule of circular parts we consider ..... circular parts.

- a) four   b) five   c) six   d) two

3. If  $u = x^2 + 2xy + y^2 + x + y$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is equal to .....

- a) 2u   b) u   c) 0   d) none of these

4. The first and second derivatives of a quadratic polynomial at  $x = 1$  are 1 and 2 respectively. Then the value of  $f(1) - f(0)$  is given by .....

- a) 3/2   b) 1/2   c) 1   d) 0

5.  $\text{curl}(xi + yj + zk) = \dots\dots\dots$

6. The value of  $\beta(1,2) + \beta(2,1)$  is .....

7.  $\iint x^2y^3 dx dy$  over the rectangle  $0 \leq x \leq 1$  and  $0 \leq y \leq 3$  is .....

8. An example of a  $3 \times 3$  matrix of rank one is .....

9. If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$  then eigen values of  $A^{-1}$  are .....

10. Let  $V$  be a vector space and  $W$  be a subspace of  $V$  then

- a)  $u+v = v+u$  for every  $u, v \in W$
- b)  $ku \in W$ , for every  $u \in W, k$  is a scalar
- c)  $m(nu) = (mn)u$ , for every  $u \in W, m$  and  $n$  are scalar
- d) All of these

### Section B

#### Short Answer Type Questions (02 Marks Each)

11. If  $u = \sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$  find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$

12. In spherical triangle  $PQR, p=62^\circ 10.1', q=111^\circ 35.2', r=63^\circ 33'$  Calculate  $P$ .

13. If  $u\vec{F} = \nabla v$ , where  $u, v$  are scalar fields and  $\vec{F}$  is a vector field, show that  $\vec{F} \cdot \text{curl } \vec{F} = 0$

14. Change the order of integration and hence prove  $\int_0^\infty \int_x^\infty \frac{1}{y} e^{-y} dx dy = 1$

15. Determine whether the set  $\{t^2+2t-3, t^2+5t, 2t^2-4\}$  of vectors is linearly independent.

### Section C

#### Answer five out of seven questions (10 Marks Each)

16. (a) Define Spherical triangle (3)

(b) In a spherical triangle  $DEF, D = 64^\circ 36', e = 90^\circ$  and  $E = 76^\circ 47'$ .

Calculate  $d, f$  and  $F$ . (7)

17. In a spherical triangle  $ABC a = 49^\circ 08', b = 58^\circ 23'$  and  $C = 71^\circ 20'$ . Calculate  $A$  and  $c$ . (10)

18. a) State Leibnitz' theorem (3)  
 b) If  $y = (\sin^{-1}x)^2$  show that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - x^2y_n = 0$ . hence find  $(y_n)_0$  (7)
19. a) If  $u = \tan^{-1} \frac{x^3+y^3}{x-y}$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$  (4)  
 b) Find the extreme points of the function  $z = x^3 + y^3 - 3axy$  (6)
20. a) Evaluate in terms of gamma function, the integral  $\int_0^\infty e^{-x^4} dx$  (5)  
 b) Find by double integration the areas enclosed by the curve  $a^2y^2 = x^3(2a-x)$  (5)
21. a) Find the directional derivative  $f = (xx^2 + y^2 + z^2)^{-1/2}$  at the point  $P(3,1,2)$  in the direction of the vector  $yx\mathbf{i} + zx\mathbf{j} + xy\mathbf{k}$  (5)  
 b) If  $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  and  $r = |\vec{r}|$  show that  $\text{div}(\frac{\vec{r}}{r^3}) = 0$  (5)
22. If  $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$  find eigen values and eigen vectors of  
 a)  $A^2 - 2A + I$     b)  $A^2$     c)  $4A^{-1}$  (10)

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