INDIAN MARITIME UNIVERSITY

(A Central University Government of India)

END SEMESTER EXAMINATION DECEMBER 2017

Programme	**:** B. Tech (Marine Engineering)	**Semester**	**:** II
Subject Name	**:** Mathematics II	**Subject Code**	**:** UG11T2202/UG11T1202
Date	**:** 19.12.2017	**Maximum Marks**	**:** 100
Time	**:** 3 Hrs.	**Pass Marks**	**:** 50

PART A

(All questions are compulsory)

Marks 10 x 3 = 30

1. a) Find a_0 in the expansion of $f(x)$ as Fourier series in the interval (-2, 2), where

$$f(x) = 0 \qquad -2 \leq x \leq 0$$

$$= 1 \qquad 0 \leq x \leq 2$$

b) Find b_n in the expansion of $f(x)$ as Fourier series in the interval $(-\pi, \pi)$ if

$$f(x) = -1 \qquad -\pi < x \leq -\pi/2$$

$$= 0 \qquad -\pi/2 < x < \pi/2$$

$= 1 \qquad \pi/2 < x < \pi$

c) Find the Laplace transform of $F(t) = 1 \quad 0 < t \leq 1$

$= t \quad 1 < t \leq 2$

$= 0 \quad t > 2$

d) Find the inverse Laplace transform of

$$\frac{3(s^2-2)^2}{2S^5}$$

e) Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ given $y(0) = 0$; $\frac{dy}{dx} = 15$ at $x = 0$

f) Find the particular integral for the differential equation

$(D^2 + 4D + 4)y = 3\sin x + 4\cos x$

g) Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$

h) A random variable *x* has the following probability function

x	-2	-1	0	1	2	3
f(x)	0.1	k	0.2	2k	0.3	k

Find the value of *K* and calculate mean and variance of the distribution.

i) If on an average 1 vessel in every 10 is wrecked, frind the probability that our of 5 vessels expected to arrive, at least 4 will arrive safely.

j) A variate x has the probability distribution

x	-3	6	9
$p(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(x),\ E(x^2)$

PART B

Answer any 5 from the following

14 x 5 = 70

2. a) Find the Fourier series expansion of $f(x) = 2x - x^2$ in (0,3) and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots - \infty = \frac{\pi^2}{12}$$ (14 marks)

3. a) Find the Laplace transform of

$$t \int_0^t \frac{e^{-t} \sin t}{t}\, dt$$

b) Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{S^2}\right)$ (6 + 8 marks)

4. a) Using convolution theorem, find $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$

b) Solve the following differential equation using Laplace transfoem

$y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$

(6 + 8 marks)

5. a) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

b) If the stream lines (paths of fluid particles) of a flow around a corner are xy = constant, find their orthogonal trajectories (called equipotential lines) (7 + 7 marks)

6. a) Solve $(D^2 - 2D + 1) = xe^x \sin x$

b) Solve by variation of parameters the following differential equation

$\frac{d^2y}{dx^2} + 4x = \tan x$

(7 + 7 marks)

7. a) In a test, an examinee either guesses, or copies or knows the answer to Multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that his answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copies is $\frac{1}{8}$. Find the probability that he knew the answer to the question given that he correctly answered.

b) Find the probability distribution of the number of green balls drawn when 3 balls are drawn one by one without a replacement from a bag containing three greens and five white balls.

(8 + 6 marks)

8. a) The probability density $p(x)$ of a continuous random variable is given by

$$p(x) = y_0 e^{-|x|} \quad -\infty < x < \infty$$

Prove that $y_0 = \frac{1}{2}$. Also find the mean and variance of the distribution.

b) A die is tossed thrice, A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.

(7 + 7 marks)

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