Indian Maritime University

(A Central University, Govt of India)

May-June 2018 End Semester Examinations

B Sc (Nautical Science)

Semester-III

Applied Mathematics-III(UG21T2302)

Duration:3 Hrs | Max Marks:70 Marks

Date: 04.07.2018 | Pass Marks:35 Marks

Note: Answer any Seven Questions out of Nine Questions.
All questions carry equal marks.

Q.1 a) Find the Laplace transform of $(\sin t - \cos t)^2$. (5 Marks)

b) i) Evaluate $L\left[\int_0^t e^{-t}\cos t \; dt\right]$

ii) Find the Laplace transform of $te^{-t}\sin 3t$. (5 Marks)

Q.2 a) Find the inverse Laplace transform of $\frac{2s-3}{s^2+4s+13}$. (5 Marks)

b) Find the inverse Laplace transform of:

(i) $\frac{1}{s(s^2+a^2)}$ (ii) $\log\frac{s+1}{s-1}$ (5 Marks)

Q.3 a) Using the Convolution theorem evaluate $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$. (5 Marks)

b) Prove that $\int_0^\infty \frac{e^{-2t}\sinh t}{t} \; dt = \frac{1}{2}\log 3$. (5 Marks)

Q.4 a) Solve the following equation by the transform method:

$y'' + y = t$, $y(0) = 1$, $y'(0) = 0$ (5 Marks)

b) Find the inverse Laplace transform of $\frac{3s+2}{s^2-s-2}$. (5 Marks)

Q.5 Solve the simultaneous equations by using Laplace transforms:

$\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$ where $x = 0$, $y = 2$ at $t = 0$. (10 Marks)

Q.6 a) Prove that $J_1''(x) = -J_1(x) + \frac{1}{x}J_2(x)$. (5 Marks)

b) Show by the use of recurrence formula:

$J_0''(x) = \frac{1}{2}[J_2(x) - J_0(x)]$. (5 Marks)

Q.7 a) Prove that $(n+1)P_{n+1}(x) = (2n+1)x\,P_n(x) - nP_{n-1}(x)$. (5 Marks)

b) Show that $\int_{-1}^{1} x\, P_n(x)\, P_{n-1}(x)\, dx = \frac{2n}{4n^2-1}$. (5 Marks)

Q.8 a) Prove that (1) $\int_{-1}^{1} [P_2(x)]^2\, dx = \frac{2}{5}$ (2) $P_n'(1) = \frac{1}{2}n(n+1)$ (5 Marks)

b) Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials.

(5 Marks)

Q.9 a) Prove that $\int x\, J_0^2(x)\, dx = \frac{1}{2}x^2\{J_0^2(x) + J_1^2(x)\}$ (5 Marks)

b) Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}$. (5 Marks)