

**INDIAN MARITIME UNIVERSITY**  
(A Central University, Govt. of India)  
**B.SC. (NAUTICAL SCIENCE)**  
**END SEMESTER EXAMINATIONS- JUNE/JULY 2019**  
**SEMESTER-I**  
**NAUTICAL MATHEMATICS**  
**(UG21T3102)**

**Date : 11.07.2019**

**Max.Marks: 70**

**Time: 3 Hrs.**

**Pass Marks: 35**

**NOTE: Question no. 1 is compulsory,**

Answer any **SIX** questions from remaining 8 questions.

All questions carry equal marks.

Scientific Calculator is permitted if required.

**1 a)** Find the  $n^{th}$  derivative of  $\log(4x^2 - 1)$ . (10marks)

**b)** If  $u = x^2y + e^{xy^2}$  prove that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ .

**c)** Express the following in the form of  $a + ib$  :  $\frac{3}{1+i} - \frac{1}{2-i} + \frac{1}{1-i}$

**d)** Compute  $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ .

**e)** State the supplementary theorem of polar triangle.

**2 a)** Evaluate  $\int_0^1 \int_{x^2}^x xy(x+y) dy dx$  (5 marks)

**b)** Express the following integral in polar coordinates and evaluate,

$\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{x^2+y^2} dy dx$ . (5 marks)

**3 a)** Evaluate  $\iiint (x+y+z) dx dy dz$  over the tetrahedron bounded by the planes  $x=0, y=0, z=0$  &  $x+y+z=1$ . (5 marks)

**b)** Evaluate  $\int_0^\infty \frac{x^5}{5^x} dx$  (5 marks)

**4 a)** In a spherical triangle PAB given , side  $a = 57^\circ$ , angle  $B = 94^\circ 01'$  and angle  $P = 71^\circ 51.5'$ . Find side  $b$  & side  $p$  by using four part formula. (5 marks)

**b)** In a spherical triangle PQR , side  $p = 73^\circ 5'$ , side  $q = 90^\circ$  and side  $r = 79^\circ 12'$ . Calculate angle P, angle Q. (5 marks)

**5 a)** In a spherical triangle LMN , *angle L = 88°24.5'* , *side n = 100°09'* , and *angle M = 97°46'* . Calculate *l , m , N*. (5 marks)

**b)** In a spherical triangle DEF , *side d = 112°3'* , *angle F = 90°* , and *angle E = 113°23'* . Calculate *angle D* and *side e*. (5 marks)

**6 a)** Prove that  $\cos^6\theta + \sin^6\theta = \frac{1}{8}(3\cos 4\theta + 5)$  . (5 marks)

**b)** Prove that  $\tan\left[i\log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2-b^2}$  . (5 marks)

**7 a)** If  $\operatorname{cosec}\left(\frac{\pi}{4} + ix\right) = u + iv$  prove that  $(u^2 + v^2)^2 = 2(u^2 - v^2)$  . (5 marks)

**b)** If  $\omega$  is a complex cube root of unity, then prove that  $1 + \omega + \omega^2 = 0$  (5 marks)

**8 a)** If  $x = \cos[\log(y^{1/m})]$  then prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (m^2 - n^2)y_n = 0. \quad (5 \text{ marks})$$

**b)** Express  $2x^3 + 3x^2 - 8x + 7$  in terms of  $(x - 2)$  by using Taylor's theorem. (5 marks)

**9 a)** If  $z = f(x, y)$  and  $x = p \cos \theta - q \sin \theta$  ,  $y = p \sin \theta + q \cos \theta$  then prove that,

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial p}\right)^2 + \left(\frac{\partial z}{\partial q}\right)^2 \quad (5 \text{ marks})$$

**b)** Find the extreme values of,

$$f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7 \quad (5 \text{ marks})$$


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