

INDIAN MARITIME UNIVERSITY
(A Central University Government of India)
END SEMESTER EXAMINATIONS-JUNE/JULY 2019
B.SC NAUTICAL SCIENCE
SEMESTER-I
NAUTICAL MATHEMATICS
(UG21T4102)

Date: 11-07-2019

Duration: 3 hrs.

Max. Marks: 70

Pass Marks: 35

Marks: 7 x 10 =70

Question 1 is compulsory. Answer any 6 questions from remaining 8 questions each of 10 marks. Scientific calculator is permitted.

1. a. Find the n^{th} derivative of $\sinh 2x \sin 4x$. (5x2=10 Marks)

b. Verify Lagrange's Mean value theorem for the $f(x) = (x - 1)(x - 2)(x - 3)$ in $(0, 4)$

c. Prove

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

d. What is the locus of $|3z - 1| = |z - 3|$

e. State supplementary theorem.

2. a. Find by double integration the area between the parabolas $y^2 = 4x$ and $x^2 = 4y$ (5 Marks)

b. Evaluate the following integral $\int_0^\infty \int_0^x x e^{-x^2/y} dy dx$ by changing the order of integration. (5 Marks)

3. a. Evaluate $\int_0^1 x^3(1 - \sqrt{x})^5 dx$ in terms of β function. (5 Marks)

- b. Evaluate $\iiint (x + y + z) dx dy dz$ over the tetrahedron bounded by the planes $x = 0, y = 0, z = 0, x + y + z = 1$. (5 Marks)
4. a. In spherical triangle PQR, sides $p = 73^\circ 5'$, $q = 90^\circ$, $r = 79^\circ 12'$. Calculate angle P . (5 Marks)
- b. In spherical triangle PVM, sides $PM = 92^\circ$, $PV = 51^\circ 55'$, and angle $V = 90^\circ$. Calculate side P . (5 Marks)
5. a. In spherical triangle PQR, sides $p = 73^\circ 5'$, $q = 90$ and $r = 79^\circ 12'$. Calculate angle P . (5 Marks)
- b. In spherical triangle ABC, angles $A = 53^\circ 5'$, $B = 129^\circ 29.3'$, $C = 55^\circ 48.8'$. Calculate side a . (5 Marks)
6. a. Examine the following function for extreme values $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ (5 Marks)
- b. If $u = e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$ (5 Marks)
7. a. If $\sin^{-1} y = 2 \log(x + 1)$, prove that (5 Marks)
- $$(x + 1)^2 y_{n+2} + (2n + 1)(x + 1) y_{n+1} + (n^2 + 4) y_n = 0$$
- b. Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$ where $u = e^{x^2 + y^2}$ (5 Marks)
8. a. If $|Z^2 - 1| = |Z|^2 + 1$ prove that Z lies in the imaginary area. (5 Marks)
- b. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$ prove that $\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}$ (5 Marks)

9. a. Expand $\cos^8 \theta$ in a series of cosines of multiples of θ (5 Marks)

b. Find $\tanh x$ if $5 \sinh x - \cosh x = 5$. (5 Marks)
