

**Indian Maritime University**  
**(A Central University, Govt of India)**  
**September/October 2024 Supplementary Examinations**  
**Programme Name: B Tech (ME)**  
**Semester: First**  
**Subject Code: UG11T4101**  
**Subject Name: Mathematics I**

Date: 02.09.2024

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

**Section A**

**Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable. 1x10=10**

1. If  $y = x^x$ , then  $y_1 = x^x \ln x + x^{x-1}$ , then  $y_2 =$

- a)  $\frac{1}{2}y_1^2 - \frac{y}{x}$       b)  $\frac{1}{y}y_1^2 + \frac{x}{y}$       c)  $\frac{x}{y}y_1^2 + \frac{y}{x}$       d)  $\frac{1}{2}y_1^2 + \frac{y}{x}$

2. If  $v = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ ,  $\frac{\delta^2 v}{\delta x^2} + \frac{\delta^2 v}{\delta y^2} + \frac{\delta^2 v}{\delta z^2}$  is

- a)  $-1/2$       b)  $1/2$       c)  $0$       d)  $1$

3. The matrix  $\begin{bmatrix} 4 & 5-4i & -2+5i \\ 5+4i & -1 & 3+i \\ -2-5i & 3-i & -8 \end{bmatrix}$  is a

- a) Skew- Symmetric matrix  
b) Symmetric matrix  
c) Skew- Hermitian matrix  
d) Hermitian matrix

4. The number of non-zero rows in an echelon form is called?

- a) rank of a matrix    b) cofactor of the matrix    c) reduced echelon form  
d) conjugate of the matrix

5. The value of the line integral  $\int_c (2xy^2dx + 2x^2ydy + dz)$  along a path joining the Origin and the point (1,1,1) is

- a) 0                      b) 2                      c) 4                      d) 6

6. A vector field which has a vanishing curl is called as \_\_\_\_\_

- a) Solenoidal field    b) Rotational field    c) Hemispheroidal field    d) Irrotational field

7. Find the curl of  $\vec{f}(x, y, z) = x^2\vec{i} + xyz\vec{j} - z\vec{k}$  at the point (2,1,-2).

- a)  $2\vec{i} + 2\vec{k}$               b)  $-2\vec{i} - 2\vec{j}$               c)  $4\vec{i} - 4\vec{j} + 2\vec{k}$               d)  $-2\vec{i} - 2\vec{k}$

8. Statement 1: The rank of  $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$  is 3.

Statement 2: The determinant of the matrix  $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$  is not equal to zero.

- a) Statement 1 is true; Statement 2 is true; Statement 2 is a correct explanation of Statement 1  
b) Statement 1 is true; Statement 2 is true; Statement 2 is not the correct explanation of Statement 1  
c) Statement 1 is true; Statement 2 is false.  
d) Statement 1 is false; Statement 2 is true

9.  $\int_0^1 \int_0^x (x^2 + y^2) dx dy =$

- a) 0                      b) 1                      c)  $\frac{1}{3}$                       d)  $\frac{1}{2}$

10. Stationary point is a point where a function f(x,y) have

- a)  $\frac{\delta f}{\delta x} = 0$   
b)  $\frac{\delta f}{\delta x} = 0$  &  $\frac{\delta f}{\delta y} = 0$   
c)  $\frac{\delta f}{\delta y} = 0$   
d)  $\frac{\delta f}{\delta x} > 0$  &  $\frac{\delta f}{\delta y} < 0$

### Section B

**Five Questions of 02 Marks each (5x2=10)**

11. Find the nth derivative of  $\frac{x}{2x^2+x-3}$ .

12. Find the angle between the tangents to the curve  $\vec{r} = t^2\vec{i} + 2t\vec{j} + 3\vec{k}$  at the point  $t=\pm 1$ .

13. Determine if the following matrix is linearly dependent or not.

$$A = \begin{pmatrix} 5 & 3 \\ 10 & 6 \end{pmatrix}$$

14. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}}$

15. Show that  $\beta \left( \frac{1}{2}, \frac{1}{2} \right) = \pi$

### Section C

**07 Questions of 10 Marks each, of which any 05 questions to be answered.**

16 a). If  $\cos^{-1} \left( \frac{y}{b} \right) = \log \left( \frac{x}{a} \right)^n$ , then prove that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$  (5)

16 b) Show that by Lagrange multiplier method if the perimeter of a triangle is constant, the triangle has maximum area when it is equilateral. (5)

17.a) If  $u = \frac{(x^2+y^2)^m}{2m(2m-1)} + x \Phi \left( \frac{y}{x} \right) + \Psi \left( \frac{y}{x} \right)$ , then prove that  $x^2 \frac{\partial^2 U}{\partial x^2} + 2xy \frac{\partial^2 U}{\partial x \partial y} + y^2 \frac{\partial^2 U}{\partial y^2} = (x^2 + y^2)^m$  (6)

17. b) If  $u = f(x, y)$ , where  $x = r \cos \theta$ ,  $y = r \sin \theta$ , then prove that  $\left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 = \left( \frac{\partial u}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial u}{\partial \theta} \right)^2$  (4)

18.a) Find the Eigen values and Eigen vectors of the matrix.

$$\begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix} \quad (7)$$

18 b) Find the rank of the matrix using row echelon form

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix} \quad (3)$$

19.a) Test for consistency and solve the equations  $5x+3y+7z=4$ ,  $3x+26y+2z=9$ ,  $7x+2y+10z=5$ . (5)

19.b) If  $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & 6 & 4+2i \end{bmatrix}$ , show that  $AA^*$  is a Hermitian Matrix where  $A^*$  is the conjugate transpose of  $A$ . (5)

20.a) A vector field is given by  $\vec{A} = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$ . Show that the field is irrotational and find the velocity potential. (5)

20b) Find the directional derivatives of the function  $f = x^2 - y^2 + 2z^2$  at the point  $P(1,2,3)$  in the direction of the line  $PQ$  where  $Q$  is the point  $(5,0,4)$ . In what direction it will be maximum? Find also the magnitude of this maximum. (5)

21. a) Prove that :  $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$  (5)

b) Change the order of integration in  $\int_0^\infty \int_y^\infty \frac{e^{-x}}{x} dx dy$  and hence evaluate. (5)

22.a) If the area bounded by  $\frac{x}{a} + \frac{y}{b} = 1$ ,  $x$  axis and  $y$  axis, rotated about  $y$  axis, then find the volume of the solid generated. (5)

22.b) Using Green's theorem, evaluate  $\int_c [(y - \sin x)dx + \cos x dy]$ , where  $c$  is plane triangle enclosed by the lines  $y=0$ ,  $x=\frac{\pi}{2}$  &  $y=\frac{2}{\pi}x$  (5)