

INDIAN MARITIME UNIVERSITY
(A Central University Government of India)
END SEMESTER EXAMINATIONS-June/July 2019
B.Tech (Marine Engineering)
Semester-I
Mathematics-I (UG11T3102)

Date: 11-07-2019

Maximum Marks: 100

Duration: 3 hrs

Pass Marks: 50

PART – A

(3 x10 = 30)

Compulsory Questions: (The symbols have their usual meanings.)

- 1. (a)** Find the n th derivative of $y = \frac{x}{(x-1)(2x+3)}$.
- (b)** If $z = e^{ax+by} f(ax - by)$ prove that $b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$.
- (c)** If $u = x^2 - y^2$, $v = 2xy$ and $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$.
- (d)** Find the radius of curvature at any point $(at^2, 2at)$ of the curve $y^2 = 4ax$.
- (e)** Prove that $\Gamma(n+1) = n\Gamma(n)$.
- (f)** Evaluate the integral $\int_0^1 \int_0^y xye^{-x^2} dx dy$.
- (g)** Find the unit normal vector to the surface $xy^2z = 3x + z^2$ at the point $(-1, -1, 2)$.
- (h)** Using Cayley Hamilton theorem find the A^{-1} of matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.
- (i)** Show that shortest distance between two points in a plane is a straight line.
- (j)** Graphically find the maximum value of $Z = 3x_1 + 2x_2$ subject to the constraints $3x_1 + x_2 \leq 15$, $x_1 + 2x_2 \leq 10$, $x_1, x_2 \geq 0$.

PART – B

(14 x 5 = 70)

Answer any FIVE of the following questions

- 2 (a)** If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0 \quad [7]$$

- 2(b)** Find the asymptotes of the curve $y^3 - 2xy^2 - x^2y + 2x^3 + 3y^2 - 7xy + 2x^2 + 2y + 2x + 1 = 0$. **[7]**

- 3(a)** If $u = \tan^{-1} \frac{x^3+y^3}{x+y}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ and

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u. \quad [3+4]$$

3(b) Examine the function $x^3 + y^3 - 3axy$ for maxima and minima. [7]

4(a) Evaluate the double integral $\iint (x + y) dy dx$ over the region bounded by $x = 0$, $x = 2$, $y = x$ and $y = x + 2$. [7]

4(b) Evaluate triple integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz dz dy dx$. [7]

5(a) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the component of the velocity and acceleration at $t = 1$ in the direction $i - 3j + 2k$. [7]

5(b) Show that the vector field $\vec{F} = (3x^2 + 3yz)\hat{i} + (3y^2 + 3xz)\hat{j} + (3xy)\hat{k}$ is irrotational. Find a scalar potential function ϕ such that $\vec{F} = \nabla\phi$. [7]

6(a) Discuss the consistency of the following system of equations and solve it if consistent.

$$x + 2y + z = 2,$$

$$3x + y - 2z = 1,$$

$$4x - 3y - z = 3$$

$$2x + 4y + 2z = 4$$

[7]

6(b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. [7]

7(a) Let $F(z) = u(x, y) + iv(x, y)$ be an analytic function of z . If $u = x^4 - 6x^2y^2 + y^4$ then find v and express $f(z)$ in terms of z . [7]

7(b) Evaluate the integral $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = 3$. [7]

8(a) Find the curve on which functional $\int_0^2 (x + y')y' dx$ with $y(0) = 0$ and $y(2) = 1$ can be extremized. [7]

8(b) Using simplex method solve the following LPP

$$\text{Maximize } Z = 5x_1 + 3x_2$$

$$\text{subject to } x_1 + x_2 \leq 2, 5x_1 + 2x_2 \leq 10, 3x_1 + 8x_2 \leq 12, x_1, x_2 \geq 0. \quad [7]$$

***** The End *****