INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

MAY/JUNE 2018 END SEMESTER EXAMINATION

B. Tech (Marine Engineering)

Semester: II

Mathematics-II (UG11T3202)

Date : 12-06-2018 MaximumMarks:100

Time: 3 Hrs Pass Marks:50

PART –A Marks: 10 x 3=30

(All Questions are compulsory)

1. (a) For any function $f(x)$ defined with in [-L,L], write down the explicit expressions of the Fourier coefficients a_n and b_n. In case of $f(x)$ being an odd function, which of these coefficients will be zero and why?

(b) If $f(t) = \sin t$ and $g(t) = \cos t$, calculate the convolution $f(t) * g(t)$.

(c) Compute L(cos^{2}t) and L(sin^{2}t) using the relation cos2t=1-2sin^{2}t=2cos^{2}t-1

(d) Calculate the inverse Laplace transform $L^{-1}\left\{\frac{5s}{s^2+2s+17}\right\}$

(e) Solve the non-exact equation $y^2dx + x(x-y)dy = 0$

(f) Find the particular integral (PI) for the differential equation

$$(D^2+3D+2)y = e^{2x}\sin x$$

(g) Derive the orthogonal trajectory corresponding to the curve $xy = C$.

(h) A bag contains five white, seven black and eight red balls. A ball is drawn at random. What is the probability that it is red ball or a white ball ?

(i) Out of 800 families with five children each, how many families would be expected to have at the most two girls ?

(j) Write down the Pearson's constants $\beta_1, \beta_2, \gamma_1, \gamma_2$ for Binomial distribution in terms of n, p, q.

PART – B

(Answer any five questions) 5 x 14=70 Marks

2 (a) Expand $f(x) = x - x^2$, $-\pi < x < \pi$ in Fourier series and show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \cdots\cdots = \frac{\pi^2}{12}$$

(b) Find the Fourier series of the triangle wave

$$f(x) = \begin{cases} \frac{\pi}{2} + x, -\pi \le x \le 0 \\ \frac{\pi}{2} - x, 0 < x \le \pi \end{cases}$$

defined on the interval $[-\pi, \pi]$.

7+7

3 (a) Find the inverse Laplace transform of $F(s) = \log\left\{\frac{s^2+4}{s^2+9}\right\}$.

(b) Use Laplace transform to solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\sin x, \quad y(0) = 0, \; y'(0) = 1$$

6+8

4 Solve the following differential equations:

(a) $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

(b) $(D^2 - 4)y = e^x + \sin 2x$

(c) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ 5+4+5

5 (a) Two fair dices are thrown simultaneously. Find the probability of getting doubles (two dices showing the same numbers) or a multiple of 3 as the sum when the sample space is assumed to be 36.

(b) An item is manufactured by three factories F_1, F_2, and F_3. The number of such items produces by the three factories are 2x, x, and x respectively. It is known that 2% of the items produced by F_1 and F_2 are defective while 4% of the items produced by F_3 are defective. All these units are put together in one stockpile and one unit is chosen at random from this stockpile. It is found that the item is defective. Calculate the probabilities of this defective unit came from F_1, F_2, or F_3. 6+8

6 (a) A discrete random variable X has the probability density function $p(x) = \frac{1}{2^x}, x = 1,2,3.....$ Find its (i) moment generating function (m.g.f), (ii) mean (μ) and (iii) variance (σ^2).

(b) A continuous normal distribution function is given by

$$f(x) = \begin{cases} ax(2-x), 0 \le x \le 2 \\ 0, elsewhere \end{cases}$$

Calculate the constant 'a' and show that the mean, mode and median of this distribution coincides to unity. 7+7

7 (a) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$$

(b) Use unit step functions and second shifting property to calculate Laplace

transform of $f(t) = \begin{cases} t^2, 0 < t < 2 \\ t-1, 2 < t < 3 \\ 7, t > 3 \end{cases}$ 8+6

8 (a) Calculate the Laplace transform for the periodic function (with period '2a')

$$f(t) = \begin{cases} t, 0 \le t < a \\ 2a-t, a \le t \le 2a \end{cases}$$

(b) When a resistance R ohms is connected in series with an inductance L henries and an e.m.f. of E volts the current i(t) is given by $L\frac{di}{dt} + Ri = E$. Calculate the current i(t) when E=10 sin(t) and the initial condition is i(0)=0.

7+7