

**INDIAN MARITIME UNIVERSITY**  
(A Central University, Government of India)

December 2016 End Semester Examinations  
B.Sc. (Nautical Science)- First Semester (2016-17 batch onwards)

Nautical Mathematics (UG21T3102)

Date : 16.12.2016

Maximum Marks: 70

Time: 3 Hrs

Pass Marks : 35

Note: Answer Any Seven Questions. All questions carry equal marks.  
Use of approved type scientific calculator is permitted.

1. a) In a spherical triangle  $PZY$ , given  $Z = 70^\circ 27'$ ,  $P = 114^\circ 54'$ ,  $Y = 109^\circ 42'$ ,  
Find  $p$  and  $z$ .  
b) In a spherical triangle  $PAB$ , given  $a = 57^\circ 00'$ ,  $B = 94^\circ 01'$ ,  $P = 71^\circ 51.5'$ ,  
Find  $b$  and  $p$  by using the four part formula. (5+5 marks)
2. a) In a spherical triangle  $ABC$ , given  $A = 100^\circ 50'$ ,  $B = 73^\circ 10'$ ,  $a = 90^\circ$ ,  
Find  $C$ ,  $c$  and  $b$ .  
b) In a spherical triangle  $PAB$ , given  $A = 111^\circ 58'$ ,  $B = 101^\circ 31'$ ,  $P = 90^\circ$ ,  
Find  $a$ ,  $p$  and  $b$ . (5+5 marks)
3. a) Prove that  $\left[ \frac{1+\sin\alpha+i\cos\alpha}{1+\sin\alpha-i\cos\alpha} \right]^n = \cos\left(\frac{n\pi}{2} - n\alpha\right) + i \sin\left(\frac{n\pi}{2} - n\alpha\right)$ .  
b) Prove that  $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$ . (5+5 marks)
4. a) Solve the equation  $7 \cosh x + 8 \sinh x = 1$  for real values of  $x$ .  
b) Prove that  $\tan \left[ i \log \left( \frac{a-ib}{a+ib} \right) \right] = \frac{2ab}{a^2-b^2}$ . (5+5 marks)
5. a) Find the  $n^{\text{th}}$  derivative of  $e^{5x} \cos x \cos 3x$ .  
b) If  $y = \sin^{-1}x$ , prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ . (5+5 marks)
6. a) Using Maclaurin's series, expand the function  $\log \sec x$ .  
b) Prove that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  if  $u = \tan^{-1} \left[ \frac{2xy}{x^2-y^2} \right]$ . (5+5 marks)
7. a) If  $u = f(e^{y-z}, e^{z-x}, e^{x-y})$  prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ .  
b) Find the maximum and minimum values of  $xy + \frac{a^3}{x} + \frac{a^3}{y}$ . (5+5 marks)

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8. a) Evaluate  $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$ .

b) Evaluate  $\iint xy(x+y) dx dy$  over the area between  $x^2$  and  $y = x$ .

(5+5 marks)

9. a) Evaluate  $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dx dy dz$ .

b) Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and the planes  $y + z = 4$  and  $z = 0$ .

(5+5 marks)

10. a) Show that  $\int_0^\infty \frac{x^4}{4^x} dx = \frac{\Gamma 5}{(\log 4)^5}$ .

b) Prove that  $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \beta(m, m)$

(5+5 marks)

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