

Indian Maritime University
(A Central University, Govt of India)

End Semester Examinations – December 2022

Programme Name: B Tech (ME)

Semester: II

Subject Code: UG11T4201

Subject Name: MATHEMATICS II

Date: 25.11.2022

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.

Section A (MCQ)

1*10=10marks

1. Which of the following is one of the criteria for linearity of an equation?
 - a) Each coefficient depends only on the independent variable
 - b) Each coefficient does not depend on the independent variable
 - c) The dependent variable and its derivatives should be of second order
 - d) The dependent variable and its derivatives should not be of same order

2. If the function $f(x)$ is odd, then which of the only coefficient is/are present?
 - a) a_0, a_n
 - b) a_n, b_n
 - c) b_n
 - d) Everything is present

3. The partial differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$; $c \neq 0$ is known as _____
 - a) Wave equation
 - b) 1-dimensional heat equation
 - c) 2-dimensional heat equation
 - d) Laplace equation

4. The solution of the differential equation $\frac{d^2 y}{dx^2} + y = 0$ is _____
 - a) $c_1 e^x + c_2 e^{-x}$
 - b) $(c_1 + c_2 x) e^{-x}$
 - c) $c_1 \cos x + c_2 \sin x$
 - d) $e^x (c_1 \cos x + c_2 \sin x)$

5. $L(e^{-3t}) = \underline{\hspace{2cm}}$

- a) $\frac{1}{s^3}$
- b) $\frac{1}{s^{-3}}$
- c) $\frac{1}{s-3}$
- d) $\frac{1}{s+3}$

6. A function $f(z)$ is analytic if _____

- a) Real part of $f(z)$ is analytic
- b) Imaginary part of $f(z)$ is analytic
- c) Both real and imaginary part of $f(z)$ is analytic
- d) None of the above

7. The value of integral $\int_C \frac{dz}{(z-1)(z-2)}$, where $C: |z| = 3$ is

- a) 0
- b) 1
- c) 2
- d) None of these

8. If the Integral Factor of $\frac{dy}{dx} + P(x)y = Q(x)$ is x , then $P(x)$ is _____

- a) $1/x$
- b) $1/x^2$
- c) x
- d) $x^2/2$

9. Which of the following is related to Cauchy Residue theorem

- a) $\int_C f(z) dz = 0$
- b) $\int_C f(z) dz = 2\pi i(\text{sum of residues})$
- c) $\int_C \frac{f(z)}{z-z_0} dz = 2\pi i f(z_0)$
- d) *none of these*

10. The inverse Laplace transform of $1/s$ is

- a) t
- b) t^2
- c) e^{-t}
- d) 1

Section B
Answer the following (5*2=10)

11. Solve $\frac{dy}{dx} = e^{2x-3y} + 2xe^{-3y}$
12. Find the orthogonal trajectories of the family of curves $y = ax^2$.
13. Find $L(t \cos t)$
14. Solve $\frac{\partial^2 z}{\partial x^2} = xy$
15. Obtain the coefficient a_0 of Fourier series for $(x) = x^2$ in the interval $0 < x < 2\pi$.

Section C

(Answer any 5 out of 7 Questions) 5*10=50 marks

16. a) Solve $(x + 2y^3) \frac{dy}{dx} = y$. **(4+6)**
b) Solve $(x^3y^2 + x) dy + (x^2y^3 - y) dx = 0$
17. a) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin(\log(1+x))$. **(5+5)**
b) Solve by the variation of parameter $y'' - 2y' + y = e^x \log x$.
18. A tightly stretched flexible string has its ends fixed at $x=0$ and $x=l$. At time $t=0$, the string is given a shape defined by $y = \mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at any time $t > 0$. **(10)**
19. **(6+4)**
a) Evaluate $L\left(t \int_0^t e^{-t} \frac{\sin t}{t} dt\right)$
b) Find the inverse Laplace transform, $L^{-1}\left(\frac{s-2}{s^3(s+2)}\right)$
20. **(5+5)**
a) Find the Fourier series for the function $f(x) = e^{-x}$ in $(0, 2\pi)$.
b) Find the Fourier series expansion for the function $f(x) = 2x - x^2$ in $(0, 3)$.
21. **(5+5)**
a) Use Laplace transform method to solve
 $\frac{d^2y}{dt^2} - 2 \frac{dy}{dt} + y = e^t$ with $y(0) = 2, y'(0) = -1$.
b) Apply Convolution theorem to evaluate $L^{-1}\left(\frac{1}{(s+1)(s^2+1)}\right)$
22. **(5+5)**
a) Find the analytic function whose imaginary part is $e^x \sin y$.
b) Evaluate $\int_C \frac{e^{2z}}{(z+i)^4} dz$, where $C : |z| = 3$

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