Indian Maritime University

(A Central University, Govt of India)

May-June 2018 End Semester Examinations

B Sc (Nautical Science)

Semester-I

Applied Mathematics I (UG21T2104)

Duration:3 Hrs | Max Marks:70 Marks

Date: 10.07.2018 | Pass Marks:35 Marks

Marks: 7 x 10 =70

Answer any SEVEN questions. All questions carry equal marks.

1. a. Find the n[th] derivative of $\cos x \; cos2x \; cos3x$

   b. If $y = a\cos(\log x) + \; b\sin(\log x)$ prove that

$$x^2 y_{n+2} + \; (2n+1)x y_{n+1} + \; (n^2+1)y_n = 0$$

(5+5 marks)

2. a. Verify Rolle's theorem for $f(x) = (x+2)^3(x-3)^4$ in $(-2, 3)$

   b. Verify Lagrange's theorem for $f(x) = \log_e x$ in $(1, e)$

(5+5 marks)

3. a. Expand $e^{\sin x}$ by Maclaurin's series upto 4 non zero terms.

   b. Find the values of a and b such that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - \; b\sin x}{x^3} = 1$$

(5+5 marks)

4. a. If $u = \tan^{-1}\left(\frac{x^3 - y^3}{x - y}\right)$, prove that $x\,\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

Hence prove $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\,\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos^3 u \sin u$

b. Find the total differential coefficient of $x^2 y$ with respect to x and y are connected by the relation $x^2 + xy + y^2 = 1$

(5+5 marks)

5. In a plane triangle, find the maximum value of $\cos A \cos B \cos C$

(10 marks)

6. a. If x and y are real, solve the equation.

$$\frac{iy}{ix + 1} - \frac{3i + 4y}{3x + y} = 0$$

b. Find the locus of z when $\frac{z-1}{z-2}$ is purely imaginary.

(5+5 marks)

7. a. Evaluate $(1 + i)^n + (1 - i)^n$

b. Evaluate $\left[\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right]^4$

(5+5 marks)

8. a. If α and β are the roots of $x^2 - 2x + 4 = 0$

prove that $\alpha^n + \beta^n = 2^{n+1} \cos\frac{n\pi}{3}$

b. If $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$ prove that

$\tan h \frac{u}{2} = \tan\left(\frac{\theta}{2}\right)$

(5+5 marks)

9. a. Expand $\cos^8 \theta$ in a series of cosines of multiples of θ

b. Also find $\cos^8 \theta - \sin^8 \theta$

(5+5 marks)
