INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

End Semester Examination Dec 2019/Jan 2020

B.Tech (Marine Engineering)

Semester -I

UG11T3102- Mathematics -I

Date: 12.12.2019 **Max Marks: 70**

Time: 3 Hours **Pass Marks: 35**

Note: i. Use of approved type of scientific calculator is permitted.
ii. The symbols have their usual meanings.

Part-A **(10x2=20 Marks)**

(All Questions are Compulsory)

1. Find the n^{th} derivative of $y = sinh2x\, sin4x$
2. Find the Radius of Curvature of at $y = e^x$ at the point where it crosses the y-axis.
3. If $u = x^2 - y^2$, $v = 2xy$ and $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$
4. If $z = u^2 + v^2$ and $= at^2, v = 2at$, find $\frac{dz}{dt}$
5. Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$ and find A^{-1}.
6. Evaluate $\int_0^{\frac{\pi}{2}} \sin^p x \cos^q x dx$
7. Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla.\vec{F}) - \nabla^2\vec{F}$
8. Prove that the shortest distance between two points in a plane is a straight line.
9. Test the analyticity of the function $f(z) = 2xy + i(x^2 - y^2)$
10. Graphically find the maximum value of $Z = 2x + 3y$ subject to the constraints $x + y \le 30$, $y \ge 3$, $0 \le y \le 12$, $x - y \ge 0$ and $0 \le x \le 20$

Part-B **(5x10=50 Marks)**

(Answer any 5 of the following)

11. a) If $y = \log\left(x + \sqrt{1 + x^2}\right)^2$ prove that $(x^2 + 1)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0$.
Hence show that $(y_{2k})_0 = (-1)^{k-1}2^k\left((k-1)!\right)^2$ where k is positive integer. **[5 Marks]**

b) Find the asymptotes of the curve
$y^3 - 2xy^2 - x^2y + 2x^3 + 3y^2 - 7xy + 2x^2 + 2y + 2x + 1 = 0$ **[5 Marks]**

12. a) If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ prove that

(i) $xu_x + yu_y = \frac{1}{2}\tan u$ **[2 Marks]**

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4cos^3 u}$ **[3 Marks]**

b) Find the maximum and minimum distances of the point $(3, 4, 12)$ from the sphere
$x^2 + y^2 + z^2 = 1$ **[5 Marks]**

13. a) Show the vector $\vec{F} = (3x^2 + 2y^2 + 1)\vec{i} + (4xy - 3y^2z - 3)\vec{j} + (2 - y^3)\vec{k}$ is irrotational and hence find the scalar potential. **[5 Marks]**

b) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the component of the velocity and acceleration at $t = 1$ in the direction $\vec{i} - 3\vec{j} + 2\vec{k}$. **[5 Marks]**

14. a) Test the consistency and solve $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ **[5 Marks]**

b) Find the latent roots and latent vectors of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ **[5 Marks]**

15. a) Evaluate $\int_0^1 \int_{e^x}^{e} \frac{dydx}{logy}$ by change of order of integration. **[5 Marks]**

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dxdydz$ **[5 Marks]**

16. a) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region a) $|z| < 1$ b) $1<|z|<2$ c) $|z|>2$ **[6 Marks]**

b) Evaluate $\int_C \frac{sin\pi z^2 + cos\pi z^2}{(z-2)(z-3)} dz$, where $C : |z| = 4$ **[4 Marks]**

17. a) Find the curves on which the functional $\int_0^1 [y'^2 + 12xy] dx$ with $y(0) = 0$ and $y(1) = 1$. **[4 Marks]**

b) Using simplex method, solve the following LPP

Maximize $Z = 4x_1 + 10x_2$ Subject to $2x_1 + x_2 \leq 50$,

$2x_1 + 5x_2 \leq 100$, $2x_1 + 3x_2 \leq 90$, $x_1, x_2 \geq 0$ **[6 Marks]**

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