

INDIAN MARITIME UNIVERSITY
(A Central University, Govt. of India)
B.SC. (NAUTICAL SCIENCE)
END SEMESTER EXAMINATIONS- JUNE/JULY 2019
SEMESTER IV
APPLIED MATHEMATICS –V
(UG21T2402)

Date: 27.06.2019

Time: 3 Hours

Max.Marks:70

Pass Marks: 35

NOTE: Attempt any **SEVEN** questions out of Nine questions.
All questions carry equal marks.

1 a) Find the analytic function whose imaginary part is

$$(x^4 - 6x^2y^2 + y^4) + (x^2 - y^2) + 2xy$$

b) Evaluate $\int_C |z| dz$ where C is the left half of the unit circle $|z| = 1$ from

$$z = -i \text{ to } z = i.$$

(5+5 marks)

2 a) Evaluate $\oint_C \frac{\sin^2 z}{(z - \frac{\pi}{6})^3} dz$ where C is the circle $|z| = 1$.

b) Determine the pole of the function $f(z) = \frac{1 - e^{2z}}{z^3}$ and find the residue at each

pole.

(5+5 marks)

3 a) Evaluate $\oint_C \tan z dz$ where C is the circle $|z| = 2$.

b) Find p if $f(z) = r^2 \cos 2\theta + i r^2 \sin p\theta$ is analytic.

(5+5 marks)

4 Show that $u = y^3 - 3x^2y$ is a harmonic function. Find its harmonic conjugate and corresponding analytic function.

(10 Marks)

5 a) Find the zeros of the following functions,

1) $f(z) = (z - 1)e^z$

2) $f(z) = z^2 \sin z$

b) A distribution consists of three components with frequencies 200, 250 and 300 having means 25, 10 and 15 and standard deviations 3, 4 and 5 respectively. Show that the mean of the combined distribution is 16 and its standard deviation is 7.2 approximately.

(5+5 marks)

- 6 The height and weights of 10 army men are given below. In which characteristics are they more variable?

Heights in cm.	170	172	168	177	179	171	173	178	173	179
Weights in kg.	75	74	75	76	77	73	76	75	74	75

(10 marks)

- 7 a) In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $2x - 9y = 107$ respectively. Calculate \bar{x} and \bar{y} and the coefficient of correlation between x and y .

- b) Ten participants in a contest are ranked by two judges as follows:-

x	1	6	5	10	3	2	4	9	7	8
y	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient ρ .

(5+5 marks)

- Q.8 a) A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
p(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

- i) Find the value of k . ii) Evaluate $P(X < 6)$, $P(X \geq 6)$.

- iii) Evaluate $P(0 < X < 5)$.

- b) The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$

If 12 such pens are manufactured, find the probability that,

- a) Exactly two will be defective.
 b) At least two will be defective.
 c) None will be defective.

(5+5 marks)

- Q.9 a) In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing

no defective, one defective and two defective blades respectively in a consignment of 10000 packets.

b) A variate X has the probability distribution :-

x	-3	6	9
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$. Hence evaluate $E(2X + 1)^2$.

(5+5 Marks)
