Indian Maritime University

(A Central University, Govt of India)
B Sc (Nautical Science)
June 2018 End Semester Examinations
Semester-I
UG21T3102 - Nautical Mathematics

Duration:3 Hrs | Max Marks:70 Marks
Date: 05.07.2018 | Pass Marks:35 Marks

Question No. 1 is Compulsory
Solve any 5 questions from remaining 7 questions.
Use of Scientific Calculator is permitted.

10 Marks

1. a. How is a spherical triangle and corresponding polar triangle connected.

   b. Find the nth derivative of $\cos x \cos 2x \cos 3x$

   c. If $Z = e^{ax+by}\ f\ (ax - by)$, prove that
   $b\frac{\partial z}{\partial x} + a\frac{\partial z}{2y} = 2abz$

   d. Evaluate
   $\int_0^{\pi/2} \sqrt{\tan\theta}\ d\theta$ in terms of gamma function.

   e. Find the locus of Z when $\frac{z+i}{z+2}$ is real.

2. a. Calculate the angles C, D and E if the sides of a spherical triangle are $c = 87°10'$, $d = 62°37$, $e = 100°10'$

   b. In spherical triangle $PVM, side\ PM = 92°$,
   side $PV = 51°55'$ and angle $V = 90°$. Calculate angle P

(8+4 marks)

3. a. If $y = a\cos(\log x) + b\sin(\log x)$

prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

b. If $u = log\left(\frac{x^4+y^4}{x+y}\right)$ evaluate $x\ \frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

(6+6 marks)

4. a. Evaluate $\int\int xy\,(x+y)dx\,dy$ over the area

between $y = x^2$ and $y = x$.

b. Evaluate $\int_0^\infty e^{-ax}\,x^{m-1}\sin bx$ in terms of gamma function.

(6+6 marks)

5. a. Expand $\cos^8\theta$ in a series of cosines of multiples of θ .

b. Find $tan\ hx$ if $5\ sin\ hx - cos\ hx = 5$

(12 marks)

6. a. In spherical triangle $DEF, angle\ D = 49°27.3'$

side $EF = 49°34'$ and $angle\ F = 100°29.6'$. Calculate sides DE, DF and angle F.

(12 marks)

7. a. Given $x + y + z = a$, find the maximum value of $x^m y^n z^p$.

b. Evaluate

$$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz\, dy\, dx$$

(6+6 marks)

8. a. If z is a homogeneous fuction of degree n in x and y, show that,

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x\, \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$$

b. If α and β are the roots of $x^2 - 2x + 4 = 0$, prove that

$$\alpha^n + \beta^n = 2^{n+1} \cos\frac{n\pi}{3}$$

(6+6 marks)
