

INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

June 2017 End Semester Examinations
B. Tech (Marine Engineering – First Semester)

Mathematics – I – UG11T 2102/UG11T 1102
(AY 2009 – 2014 onwards)

Date: 29.06.2017

Maximum Marks : 100

Time: 3 Hrs

Pass Marks : 50

- Note:**
- i. Use of approved type of scientific calculator is permitted.
 - ii. The symbols have their usual meanings.

Part-A

(10x3=30 Marks)

(All Questions are Compulsory)

- Q.1
- (a) Prove that the shortest distance between two points in a plane is a straight line.
 - (b) Find radius of curvature at (1, -1) of the curve $y = x^2 - 3x + 1$.
 - (c) Find the saddle points of the function $x^3 + 3xy + y^3$.
 - (d) Find the n^{th} derivative of $\frac{x}{(x+9)(2x+5)}$.
 - (e) Show that $\vec{F} = yz \hat{i} + xz \hat{j} + xy \hat{k}$ is the solenoidal and irrotational vector field.
 - (f) Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex (0, 0) to one extremity of latus rectum (a, 2a).
 - (g) Prove that $[\vec{a} \quad \vec{b} + \vec{c} \quad \vec{a} + \vec{b} + \vec{c}] = 0$
 - (h) Determine the poles of the function $f(z) = \frac{z^2}{(z+1)^2(z-2)}$ and the residue at each pole.
 - (i) Prove that $\sqrt{n+1} = n\sqrt{n}$
 - (j) Evaluate $\int_0^1 \int_0^y xy e^{-x^2} dx dy$

Part-B

(5x14=70 Marks)

(Answer any 5 of the following)

Q.2 (a) If $y_n = \frac{d^n}{dx^n} (x^2 - 1)^n$, show that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$

(b) Find the asymptotes to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

(7 + 7=14 Marks)

Q.3 (a) If $u = \log (x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x+y+z)^2}$$

(b) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

$$\text{and } x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4 \cos^3 u}$$

(7 + 7=14 Marks)

Q.4 (a) Investigate for what values of K and W the simultaneous equations

$$x + y + z = 6; \quad x + 2y + 3z = 10; \quad x + 2y + Kz = W$$

have (i) no solution,

(ii) a unique solution ,

(iii) an infinite number of solutions.

(b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 9 & -8 \\ 5 & -4 \end{bmatrix}$

(8 + 6= 14 Marks)

Q.5 a) Find the length of arc of the curve $x = ae^t \text{Sint}$, $y = ae^t \text{Cost}$ from $t=0$ to $t=1$.

b) Apply the rule of differentiation under integral sign to prove

$$\int_0^{\pi/2} \frac{\log(1 + a \sin^2 x)}{\sin^2 x} dx = \pi(\sqrt{1+a} - 1) \quad (7 + 7=14 \text{ Marks})$$

Q.6 a) Find the volume of the solid generated by the revolving region bounded by $y = x^3$, $y = 0$ and $x = 2$ about the x-axis.

b) Evaluate $\iiint_V dx dy dz$ where V is the volume bounded by cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$, $z = 0$.

(8 + 6=14 Marks)

Q.7 a) Find the directional derivative of $\phi = xy^2 + yz^2$ at $(1, -1, 1)$ along the vector $\hat{i} + 2\hat{j} + \hat{k}$.

b) If the vector field $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational, find the values of a, b, c to determine the scalar function ϕ such that $\vec{F} = \nabla\phi$.

(7 + 7=14 Marks)

Q.8 a) Evaluate $\oint_C \frac{z+3}{(z-1)^2(z+2)} dz$ where C is the circle $|z|=3$

b) Let $f(z) = u + i v$ be an analytic function and if $u = -3x + 2xy$, then find v and express f(z) in terms of z. (7 + 7=14 Marks)
