INDIAN MARITIME UNIVERSITY

(A Central University, Government of India)

December 2016 End Semester Examinations
B.Tech. (Marine Engineering) First Semester

Mathematics - I (UG11T1102/ UG11T2102)

Date : 16.12.2016 **Maximum Marks: 100**
Time: 3 Hrs **Pass Marks : 50**

Section – A (10 x 3 = 30 Marks)

Compulsory Questions: (The symbols have their usual meanings)

1.(a) If $y = \sin^4 x$ then find y_n.

(b) If u = f (r , s , t) where $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$ show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$

(c) In an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ prove that $\rho = \frac{a^2b^2}{p^3}$ where ρ = radius of curvature at any point (a cos t , b sin t) of the ellipse and p=central perpendicular on the tangent at (a cos t , b sin t).

(d) Show that in the catenery $y = c\cosh\frac{x}{c}$

(i) the length of the arc being measured from the vertex to any point is given by

$$s = c\sinh\frac{x}{c}$$

(ii) $y^2 = c^2 + s^2$, the arc being measured from the vertex.

(e) Evaluate $\int_0^a \int_0^x \int_0^y x^3 y^2 z\,dz\,dy\,dx$

(f) Find the eigen values of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

(g) Find the directional derivative of $\phi(x,y,z) = x^2y^2z^2$ at the point (1,1,-1) in the direction of the tangent to the curve $x = e^t, y = 2\sin t + 1, z = t - \cos t$ at $t = 0$.

(h) Find $\phi(r)$ such that $\vec{\nabla}\phi = \frac{\vec{r}}{r^4}$ and $\phi(1) = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

(i) Evaluate $\int_C \frac{3z^2 + z}{z^2 - 1} dz$ where c is the circle $C: |z-1| = 1$.

(j) Test for an extremum of the functional

$$I[y(x)] = \int_0^1 (xy + y^2 - 2y^2y')dx, y(0) = 1, y(1) = 2$$

Section –B (5 x 14=70 Marks)

Attempt any five questions from the following:

2.(a) If $y = [x + \sqrt{1 + x^2}]^m$ then prove that

(i) $(1 + x^2)y_2 + xy_1 - m^2y = 0$

(ii) $(1 + x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

(iii) Find $(y_n)_0$, where $(y_n)_0$ means the value of the nth derivative of Y when x = 0.

(b) Find the asymptotes to the curve

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$$

(8 + 6)

3. (a) Find a point in the plane x + 2y + 3z = 13 nearest to the point (1,1,1) using the method of Lagrange's multipliers.

(b) If $z = x^n f(\frac{y}{x}) + y^{-n}\phi(\frac{x}{y})$ show that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = n^2 z$$

(7 + 7)

4.(a) Find the volume and surface area of the solid formed by the revolution of the cycloid $x = a(\theta + \sin\theta), y = a(1+\cos\theta)$ about its base.

(b) Apply the rule of differentiation under the sign of integration to evaluate the integral

$$\int_0^\infty e^{-x^2 - \frac{a^2}{x^2}} dx$$

(7 + 7)

5.(a) Evaluate the integral $\iint_R (1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}) dxdy$ where R is the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant.

(b) Change the order of integration and hence evaluate $\int_0^1 \int_{e^x}^{e} \frac{dxdy}{y^2 \log y}$

(8 + 6)

6.(a) Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$

(b) Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find a scalar function ϕ such that $\vec{A} = \nabla\phi$.

(6+8)

7.(a) Using Caley-Hamilton theorem find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{bmatrix}$$

(b) Apply the rank test to examine if the following system of equations is consistent and if consistent then find the complete solution :

2X + 3y +4 z 11, x +5y +7z = 15, 3x + 11y + 13z = 25 (8 + 6)

8.(a) Determine the poles of the following function and residue at each pole :

$$f(z) = \frac{z^2}{(z-1)^2(z+2)}$$

And hence evaluate $\int_c \frac{z^2 dz}{(z-1)^2(z+2)}$ where c: $|Z| = 3$

(b) Expand $f(z) = \frac{1}{(z-1)(z-2)} : 1 < |z| < 2$ in Taylor's and Laurent's series.

(10+4)

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